A BRIEF HISTORY OF SPACE-TIME*

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Abstract?

1 Introduction

Up to the present, all fundamental[a] physical theories have involved space-time structures. That is, such theories have involved certain concepts of space and time, related to each other by the concept of change of something[b] in space through time. Traditionally, geometry was a metrical description of three-dimensional spatial intervals[c]; similarly, a metrical description of temporal intervals is called chronometry; together, they constitute the chronogeometrical structure of space-time. The concept of change may either refer to motion, that is changes of place of some limited region of space over time, as in the motion of a point particle; or to quantitative changes of some quality (or qualities) attached to each place in a region of space over time, as in field theories[d].

The analysis of motion generally presupposes a class of natural (or force-free) motions; any deviation from such motions is attributed to the action of a force. Mathematically, the class of natural motions is described mathematically with the help of an *inertial connection* over space-time. As discussed in more detail below, the deeper meaning of the equivalence principle is that the effects of gravitation may be considered not as due to a "gravitational force",

*MY TITLE IS GIVEN, OF COURSE, IN HUMOROUS HOMAGE TO STEVEN HAWKING'S *BRIEF HISTORY OF TIME*. THOSE LOOKING FOR AN ACTUAL BRIEF HISTORY OF THE CONCEPT ARE REFERRED TO STACHEL 1999BCITE.

[a] I add "fundamental" to exclude phenomenological theories such as thermodynamics, which do not involve space-time structures.

[b] Describing just what that something is, is the task of each physical theory.

[c] Currently any mathematical description of a space of any number of dimensions is called a *geometry*. See the following section.

[d] For example, in a perfect fluid theory, the qualities attached to each point of the fluid field would be the density and pressure of the fluid. In electromagnetic field theory in the (3+1)-dimensional formulation, the qualities would be the electric and magnetic fields, described by two three-vectors, at each point of the electromagnetic field.

as Newton originally suggested; but as part of a class of natural motions described by an *inertio-gravitational connection*[e].

Taken together, the chrono-geometrical and inertio-gravitational structures of any physical theory constitute what I call its *space-time structures.* I shall present a brief historical-critical[f] survey of the space-time structures that characterize the development of theoretical physics from Newton to Einstein, i.e., the role in these theories of the chrono-geometrical and inertio-gravitational structures and of the compatibility conditions between them. I shall then emphasize the ways in which the space-time structure of the general theory of relativity, by virtue of its dynamization of the chrono-geometry, differs radically from that of all previous physical theories. I shall stress two fundamental differences:

1. the basically local nature of the theory implies the absence of a preassigned global manifold; the local spatio-temporal structures of particular solutions to the field equations determine the global manifold;

2. the role of diffeomorphism invariance in precluding the existence of any pre-assigned (kinematical) spatio-temporal properties of the points of the manifold (even locally) that are independent of the choice of a solution to the field equations (no kinematics before dynamics). The physical points of space-time thus play a secondary, derivative role in the theory, and cannot be used in the formulation of physical questions within the theory (they are part of the answer, not part of the question).

Finally, I shall discuss the extent to which these features can be generalized, and (assuming that the generalized features are characteristic of the problem) the challenge that they present to any theory of quantum gravity. But before I go into these matters, I shall discuss some needed mathematical preliminaries.

2 Geometry and Algebra: Points and Coordinates

We may define a *geometry* as the study of a set S of elements p, denominated *points* without prejudice as to their nature, together with a set R of *basic*

[e] A connection (inertial or inertio-gravitational) is also needed in field theories: it allows us to compare the quantitative changes of some quality at neighboring points of space-time.

[f] I use this term in homage to Ernst Mach, who so characterized his surveys of the history of mechanics, optics and thermodynamics. It also serves to emphasize that I am not attempting a true historical survey, but rather a retrospective account based on our current understanding of these theories.

relations between these points. The points are *homogenous* in nature, at least insofar as concerns the geometrical relations between them[g]. This means that there exists a group g of *automorphisms* (one-one mappings of the set of points onto itself $p \leftrightarrow p'$) that leaves all the basic relations undisturbed (see Weyl 1946[24], Weyl 1949[25]): if $R(x, y, z \ldots)$ holds, where x, y, z are free variables that may take any (distinct) points as their values and $x \to x'$, $y \to y'$, etc., under the automorphism, then $R(x', y', z' \ldots)$ also holds. And such g acts *transitively*: if p and q are any two points of the set, there exists at least one automorphism taking p into q. Alternatively (Klein 1872[10]), one may define a geometry by the set of elements and an abstract group G, which has a faithful realization in the group g of automorphisms that acts transitively on S. Relations between the points are geometrical iff they are invariant under all automorphisms[h].

Basic to progress in both the mathematical treatment of geometries and their applications in physics is the process of *coordinatization* (Weyl 1946[24], Shafarevich 1992[?]); that is, the assignment of a *frame*, consisting of a set of "objectively individualized reproducible" *coordinates*, quantities in the most general sense, which are put into one-one correspondence with the homogeneous elements (points) of the geometry[i]. The use of the term "quantity" by no means implies that the coordinates making up the frame must be ordinary numbers. While they must share with the latter the property of being inherently distinguishable from each other, they may be any sort of quantities (ordinary Cartesian coordinates, non-commuting coordinates, quaternions, vectors, matrices, etc.) that are adapted to the description of the relations constituting a particular geometry. Indeed, the construction of these quantities and the study of their properties has been said to constitute the

[g] "There is no distinguishing objective property by which one could tell apart one point from all the others; fixation of a point is possible only by a demonstrative act as indicated by terms like 'this', 'here'." (Weyl 1946[24], p. 14).

[h] "The mathematician unwilling to draw on any external truth will be inclined to take the view that any group whatsoever can be appointed as the group of automorphisms; he declares by this appointment or convention that he is going to study only such relations among points as are not destroyed by mappings of his group" (Weyl 1946[24], p. 15).

[i] "The meaning of coordinatisation is to specify objects forming a homogeneous set X by assigning individually distinguishable quantities to them. Of course, such a specification is in principle impossible: considering the inverse map would then make the objects of X themselves individually distinguishable. The resolution of this contradiction is that, in the process of coordinatisation, apart from the objects and quantities, there is in fact always a third ingredient, the coordinate system (in one or another sense of the word), which is like a kind of physical measuring instrument. Only after fixing a coordinate system S can one assign to a given object x∈X a definite quantity, its 'generalised coordinate'." (Shafarevich 1992[?], p. 160).

domain of *algebra*[j].

The introduction of a frame and coordinates introduces a "fundamental problem ... how to distinguish the properties of the quantities that reflect properties of the objects themselves from those introduced by the choice of a coordinate system? This is the *problem of invariance* of the various relations arising in theories of this kind. In spirit, it is entirely analogous to the *problem of the observer* in theoretical physics" (Shafarevich 1992[?], p. 160). The answer to this problem is that there must be a realization of the automorphism group that acts on the frames, transforming one frame into another. "If the set of quantities forms a vector space, then this action defines a representation of G [the automorphism group]" (*ibid.*) The realization (resp. representation) must be frame-independent, assuring the objectivity of the entire class of frames and corresponding coordinate systems (Weyl 1946[24], 1949[25]).

3 Space-Time Structures

Traditionally, space-time structures have been associated with the behavior of ideal measuring rods (*geometry*) and clocks (*chronometry*) and "free" test-particles (*inertial structure*)[k]. With the inclusion of gravitation in the picture, it has been found possible to place it on the inertial side of the equation of motion of a "free" particle, both at the Newtonian and Einsteinian levels, so that we shall then speak of the *inertio-gravitational structure*, represented mathematically by a symmetric affine connection. The special theory of relativity showed the need to combine the 1-dimensional chronometrical and 3-dimensional geometrical structures into one 4-dimensional *chrono-geometrical structure*, represented mathematically by a pseudo-metrical tensor field; but we can and shall adopt the 4-dimensional point of view even for the pre-relativistic Galilei-Newtonian theories, in which the geometry is represented by a degenerate contravariant metric field of rank three[l], and the chronometry

[j] "[W]hen we meet a new type of object, we are forced to construct (or to discover) new types of 'quantities' to coordinatise them. The construction and the study of the quantities arising in this way is what characterises the place of algebra in mathematics (of course, very approximately). From this point of view, the development of any branch of algebra consists of two stages. The first of these is the birth of the new type of algebraic objects out of some problem of coordinatisation. The second is their subsequent career, that is, the systematic development of the theory of this class of objects..." (Shafarevich 1992[?], p. 8)

[k] Our point of view is that the use of such ideal elements (measuring rods, clocks and free test particles) does not amount to an "operational definition" of the quantities they measure; rather it amounts to an assertion of the existence (relative to the theory in question) of the structures (geometrical, chronometrical and inertial, respectively) being measured.

[l] While this is how it is usually introduced, it is actually more intuitive to introduce a triad of mutually orthogonal unit vector fields, representing a frame of reference that allows us

by a scalar field (the absolute time), whose gradient has a vanishing transvection with the metric field. *Compatibility conditions* between the chronogeometrical and the inertial or inertio-gravitational structures essentially require that freely falling clocks and measuring rods still remain good instruments; and that the proper time of a clock travelling along such an inertial path also measures the affine time along that path. Mathematically these amount to the conditions that the metric field and the gradient of the absolute time have vanishing covariant derivatives with respect to the inertio-gravitational connection.

To sum up the previous section, by its very definition, a geometry consists of elements (points) that are homogeneous under some group of automorphisms; in order to develop any sort of calculus for the treatment of a geometry, a frame of reference must be adopted that allows us to individuate the points by a coordinatization. So much is common to all geometries. But when we wish to discuss space-time geometries another important distinction arises between those space-time geometries that allow a *kinematical coordinatization* and those that do not, but require a *dynamical coordinatization.* I shall discuss particular space-times in the next section; but shall preface this discussion with some general remarks that will hopefully make the distinction clear(er). We may imagine a physical theory to consist of two distinct sets of relations: one set is fixed and given; while the second set of relations is widely variable but subject to certain restrictions, which we shall call *dynamical.* If the fixed set of relations determines the group of automorphisms of the space-time associated with the theory, we shall call the relations *kinematical*; and the class of frames and coordinates singled out by these automorphisms *kinematical frames and coordinates*[m]. If, on the other hand, the theory has no fixed set of relations, so that any physical frame and coordinatization must be associated with a set of dynamically determined relations, we shall call them *dynamical frames and coordinates.*

4 Historical-Critical Survey

We shall eschew discussion of pre-Newtonian theories and confine ourselves to discussion of the space-time structures associated with four theoretical

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to coordinatize the Euclidean hyperplanes on which the absolute time is constant.

[m]We must also allow for the possibility that, although the kinematical frames and coordinates *could* be defined independently of dynamical relations, in point of fact they are actually defined with the help of dynamical relations; so long as the resulting coordinatization is *independent* of the particular dynamical relations chosen, it is still kinematical in nature.

Figure 1. Caption?

frameworks:

1. Galilei-Newtonian space-time;
2. Minkowskian space-time;
3. general non-relativistic space-times;
4. general-relativistic space-times.

Note that 1) and 2) refer to individual space-times; while 3) and 4), which include gravitation at the Newtonian and Einsteinian levels, respectively, refer to classes of space-times picked out by dynamical equations for the inertio-gravitational field.

Before going into more detail, let me situate these theories with respect to a cube that is even more fascinating — at least for a physicist — than the Rubik cube. I shall call it the “”Bronstein cube” since it is based on an idea introduced by the Soviet physicist Matvey Petrovich Bronstein (see Bronstein 1933[?]; Einstein 1999a[?]). The axes of this cube (see Figure 1) are formed by the three fundamental constants c, G, and h. We start at the origin with Galilei-Newtonian space-time and the corresponding non-gravitational physics (classical mechanics). If we move in the G-direction, we reach general non-relativistic space-times (the name is due to Ehlers 1973a[5], b[6]), in which Newtonian gravitational theory is interpreted as a transformation of the fixed inertial structure of Galilei-Newtonian space-time into a dynamical inertio-gravitational structure. If we move in the c-direction, we reach Minkowski space-time, with its modification of the Galilei-Newtonian chronogeomery, and special-relativistic physics (relativistic mechanics, Maxwell's electrodynamics, etc). If we move simultaneously in the c-and G-directions, i.e., in the $c-G$-plane, we reach general-relativistic space-times, with their modifications of both the chrono-geometrical and inertio-gravitational structures. If we move from the origin in the h-direction, we reach non-relativistic quantum mechanics; while if we move simultaneously in the c-and h-directions, i.e., in the $c-h$-plane, we reach (special-relativistic) quantum field theories. If we move simultaneously in the G-and h-directions, i.e., in the $G-h$-plane, we reach the quantized version of Newtonian gravitational theory. And of course, if we move simultaneously in all three directions, i.e., into the cube, we reach the promised land: quantum gravity.

Figure 2. Caption?

It is not clear to what extent the operations of moving in a plane and then adding the third dimension, commute: in particular if we start from quantum field theory ($c-h$-plane) and try to reach the promised land by adding G, it is not at all obvious that we shall reach the same place that we reach if we start from general relativity ($c-G$-plane) and add h. Indeed, we might characterize string theory as the result of an attempt of the first type, and loop quantum gravity as the result of an attempt of the second type[n]. I shall proceed on the assumption that general relativity contains some important lessons about the structure of space-time that should not be ignored in the search for a future quantum theory of gravity, and discuss the "Bronstein ($c - G$) plane", and the lessons that can be drawn from it about how to move in the h-direction.

Now let us consider the four classes of space-times in the Bronstein plane (see Figure 2) in a little more detail:

1. Galilei-Newtonian space-time has the inhomogeneous Galilei group as its group of automorphisms; it consists of three spacelike and one time-like translation, three spatial rotations and three Galilei boosts. The chronometrical, the geometrical and the inertial structures are all non-dynamical.

2. Minkowski space-time has the Poincaré (inhomogeneous Lorentz) group as its group of automorphisms. While the translations and rotations are similar, the boost are now Lorentz boosts. Chronometry and geometry are fused into a single chrono-geometrical structure; but it and the inertial structure remain non-dynamical.

3. General non-relativistic space-times have an automorphism group that has been called the Newtonian group (see Ehlers 1973a[5],b[6]); in addition to the translations and rotations, it includes transformations between all rigid, non-rotating frames of reference. So it is no longer a finite-parameter Lie group but, in addition to the spatial translations and rotations and time translations, involves "boosts" (now including arbitrary rigid linear accelerations) that depend on three functions of one variable

[n] Lee Smolin pointed out that in fact loop quantum gravity actually originated from consideration of loop quantizations of special-relativistic quantum field theories. But the current diffeomorphism-invariant loop quantization procedure in quantum gravity is logically independent of its historical origins. See, e.g., Rovelli 1998[11], section IIB.

— the absolute time. Even though its definition involves the dynamical inertio-gravitational structure, the resultant structure is independent of the particular dynamical field chosen; so that choice of a frame and associated coordinatization is still kinematical in nature. Thus, while the inertio-gravitational structure is dynamical, the chronometry and geometry remain non-dynamical.

4. general relativistic space-times have the diffeomorphism group as their automorphism group. Both the chrono-geometrical and inertio-gravitational structures are dynamical.

While they differ in a number of ways, the first two of the above classes of space-times have several features in common, by which they differ from the last two. Most important for our purposes, their automorphism group are both finite-parameter Lie groups. In both cases, this implies that we can single out physically a class of inertial frames, each of which can serve as the physical basis of one of the coordinate frames discussed earlier. An inertial frame can be physically realized by singling out ("pointing to" in Weyl's terminology) some body in rigid inertial (force-free) motion. (I emphasize that, within the two theoretical frameworks we are discussing, this is a kinematical concept, involving only the geometry, chronometry, and inertial structures of space-time.) All we need to add is a choice of original on the body selected, a set of three mutually perpendicular directions, and a unit of length (i.e, another point on the body besides the origin). The usual Cartesian spatial coordinatization can thus be physically realized. For Galileian space-time, the coordinatization of the absolute time can now be fixed by a choice of temporal origin and a unit of time. For Minkowski space-time, the relativity of simultaneity presents an additional complication; but, as I have shown elsewhere (see Stachel 1983[14]), we do not even need to introduce clocks as separate entities: they may be constructed from families of bodies in inertial motion relative to each other, and the Poincaré-Einstein simultaneity convention can also be defined in terms of these bodies. In both cases, we now have a kinematical coordinatization of the space-times.

By way of pointing out the sharpest contrast, let us jump to case 4), general relativistic space-times. Here, no physical significance can be attached to a coordinatization that is independent of the dynamical fields, and in particular of the pseudo-metric tensor field which is always present[o]. For let us assume the contrary, i.e, suppose that such a kinematical coordinatization

[o]We can always formally introduce a coordinatization via a fibration and foliation of the manifold; but unless these can be associated in some way with some dynamical fields, such a mathematical coordinatization has no physical significance.

were possible. Then, as the well-known hole argument shows, the theory could not have a well-posed Cauchy problem[p]. Indeed, more generally, given any solution to the field equations on the manifold, we could always construct another solution that differs from it only within some open set H (the hole). To see this, introduce any diffeomorphism that is equal to the identity on the complement of the open set, M-H; but differs from the identity on H. Then, by the nature of generally-covariant field equations, the carry-along of the original solution by this diffeomorphism is also a solution to these field equations; and indeed a solution that differs from the original one only on the open set. Taking H to be the future of any space-like Cauchy surface, we see that we can apparently construct two (and by extension a four-fold functional infinity) of solutions with the same initial data by choice of the diffeomorphism on H. If the points of the manifold were physically distinguished *kinematically* (i.e., independently of the solutions to the field equations), we should have to regard these solutions as physically distinct. The only ways out of this dilemma (one, by the way that both Einstein *and* Hilbert originally confronted) are to break general covariance by attaching physical significance to four additional non-covariant equations (cf. Fock, Luganov, etc.); or to accept that only a dynamical physical coordinatization is possible in a generally covariant theory.

By way of contrast, let me highlight the difference by explaining why the hole argument fails for space-time theories such as cases 1) and 2) In both of these cases, if the group of automorphisms is such that, if an element of it reduces to the identity on the complement of an open set, then it reduces to the identity everywhere; so the hole argument is clearly blocked[q]. That is certainly the case if the automorphism group is a finite-parameter Lie group, which is just the case for the inhomogeneous Galilei (case 1) and Lorentz groups (case 2). Case 3), the Newtonian group, is a little more complicated. If the hole H is finite in size, then indeed an element of the automorphism group is fixed by its values on M-H, since its value is thereby fixed for all

[p]For the hole argument, see Stachel 1989[16], pp. 71-81, Einstein 1993a[?]. By way of contrast with the space-time manifold, consider the space of perceived colors. This is a three-dimensional manifold, the points of which are coordinatized perceptually by three coordinates, for example: hue, brightness and saturation, quite independently of any metric that may be imposed on the space. See Stachel 1989[16], p. 76; and for a recent discussion of the geometry of color space Ashtekar, Corichi and Pierri 1999[1].

[q]An attempt has been made to extend the hole argument to all space-time theories (see Earman and Norton 1978[?]) by rewriting them in generally covariant form. But of course this does not change the automorphism group of the space-time theory. If the theory is written in generally covariant form, geometrical object fields have to be explicitly introduced to describe the dynamically-independent chronometrical, geometrical (or chrono-geometrical) and inertial structures of space time. The automorphisms of the space-time geometry are then defined as the subclass of diffeomorphisms that are *symmetries* of these fields.

times. But if H is the future of any (spacelike) hypersurface $t_(0)$=const, then we can change the value of the inertio-gravitational field by an automorphism of the Newtonian group that reduces to the identity before $t_(0)$, and is a linear acceleration transformation after $t_(0)$. But this is just what we expect physically, since the equivalence principle asserts the inability to distinguish between accelerations of frames and (a certain class of) changes of the inertio-gravitational field; and mathematically, since the inertio-gravitational field is not a tensor field but a connection field, the components of which need not disappear in all frames of reference just because they do in one.

Is it a bad thing that kinematical coordinatizations are impossible in general relativity? Einstein certainly did not think so.

> You consider the transition to special relativity as the most essential thought of relativity, not the transition to general relativity. I consider the reverse to be correct. I see the most essential thing in the overcoming of the inertial system, a thing that acts upon all processes but undergoes no reaction. This concept is, in principle, no better than that of the center of the universe in Aristotelian physics. ...Contemporary physicists do not see that it is hopeless to take a theory that is based on an independent rigid space (Lorentz-invariance) and later hope to make it general relativistic (in some natural way)[r].

The dynamizing of the inertio-gravitational structure enables the elimination of (global) inertial frames, those unmoved movers that influence the rest of physics without themselves being affected by any physical processes. What about the chronogeometry? If one were to try to preserve its special relativistic form, one would have to give up the compatibility conditions, and regard the inertio-gravitational field as exerting a distorting influence on the behavior of (ideal) measuring rods and clocks, so that the "measurable but distorted" chrono-geometry, would differ from the "true but hidden" one. This is how many quantum field theorists in effect interpret general relativity. But I insist this is actually a different interpretation of the same field equations, *not* equivalent to general relativity — and ultimately untenable, not least because of the hole argument[s].

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[r]Einstein to George Jaffe, 19 January 1954. Cited from Stachel 1987[15], p. 201.

[s]To add just a few salient comments on this point: The global topology of a solution to the general relativistic field equations may not be the same as that of the Minkowski space-time manifold. Assuming that it is, there is no unique mapping of the points of the two manifolds onto each other. Indeed, due to the diffeomorphism group, there is a four-fold functional ambiguity in any such mapping. At the level of attempts at perturbative quantization starting from Minkowski space-time, this ambiguity manifests itself in the fact that the

5 Lessons for the Future?

The hole argument may be generalized in several directions[t]. In particular, one can apply a forgetful functor to the differentiable manifold with its diffeomorphism group of mappings between points, getting a set with its permutation group of mappings between elements. The geometrical object fields can then be replaced by functions of (or relations between) the elements of the set. If we require that:

1. any theory that picks out a class of functions of (or relations between) the elements of the set be generally permutable; and
2. it must be possible to specify a model of the theory without giving the values of the functions at (or the applicability of the relations to) every element of the set;

then it follows from this generalized version of the hole argument that the elements of the set can have no individuating properties that they do not inherit from a chosen model of the theory. This result will apply to all discrete structures, such as causal sets, that may be introduced to model or replace space-time[u].

One can also generalize in other directions, in which the coordinates introduced to represent the salient features of the geometry are not ordinary numbers, such as non-commuting geometries. As long as the automorphism group of the geometry is a function group rich enough to include elements not uniquely fixed by their action on the points of M-H, the hole argument will go through, and if we want a theory such that a model does not have to be specified by giving the values of the relevant functions or relations at each and every point of the manifold (or what have you), then all distinguishing or individuating features of the points must depend on the model chosen.

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first order "gauge transformations" can also be interpreted as first order diffeomorphisms: when we write $g_(\mu\nu) = \eta_(\mu\nu) + h_(\mu\nu)$, it is customary to consider $h_(\mu\nu) + \xi_(\mu;\nu) + \xi_(\nu;\mu)$ as a gauge-transfomed version of the same h field on the given background Minkowski space-time. But it is just as valid to associate the Killing form with the Minkowski metric: $\eta_(\mu\nu) + \xi_(\mu;\nu) + \xi_(\nu;\mu)$; and regard it as a diffeomorphism of the Minkowksi space-time. As a solution to the Einstein equations, Minkowski space-time is just as diffeomorphism-invariant as any other solution. Within Einstein's theory, diffeomorphism invariance is inescapable; and if one insists on escaping it and maintaing only Poincare-invariance, one is no longer within Einstein's theory, even if the field equations are formally the same.

[t] For more details, see Stachel 2002a[21], 2002b[22].

[u] Although up to now all fundamental theories have involved space-time structures, it by no means follows that this will continue to be so in the future. Space-time may turn out to be a derived concept in some future fundamental theory. Indeed, Einstein had already considered such a possibility: see Einstein 1993a[?] for details.

The common feature of all these cases is that they result in relational theories, in the sense that there are no features of the underlying points that do not result from the ensemble of relations imposed on them by the particular model of the theory in question: aside from the underlying set and its permutation group, there are no universal, model-free structures. Is physics likely to proceed in the direction of such relational theories?

No one can say for sure, of course. But for some of us, the lesson of general relativity is very persuasive. Once one has given up the idea of the existence of a space-time structure that is independent of dynamics, and that consequently can be coordinatized in a dynamics-independent way; once one has realized that we are not forced to introduce such kinematical structures into our physical theories (such as inertial frames of reference) that affect all the rest of physics (i.e., all dynamical theories) but are themselves unaffected by the nature of these theories in general or in particular by the solution to the dynamical field equations that is adopted, it is hard to believe that the development of physics will force us back to the use of such unmoved movers. However we may be led to generalize the nature of the points that make up our space-time — or whatever takes its place; however we may generalize the algebra of the coordinates used to describe these points, as long as the points are only individuated dynamically, i.e., by means of the solutions to some dynamical equations limiting the coordinates — even if we replace the space-time structures by some structures we come to regard as more fundamental, we may hope that we shall only be advancing further along the road charted by general relativity.

Quantum mechanics is based on the paradigmatic use of the concept of a probability amplitude $< a, x_(1), t_(1)|b, x_(2), t_(2) >$. This may be interpreted physically in form of the answer to a question: If something (a) is prepared (and registered) here-and-now (x1,t1), what is the probability of something else (b) being detected (and registered)[v] there-and-then (x2,t2)? (The answer of course is the amplitude squared.) It depends on our being able to know just what here-and-now and there-and-then signify, so that we can place our preparation and detection apparatus at the right places and activate them at the right times (with respect to some inertial frame of reference) in order to

[v] As Bohr emphasized, a quantum mechanical phenomenon always includes the acts of preparation and detection, which each involve some irreversible change (which I call a regristration (?????)) in some physical system. (Feynman's concept of a physical process is very similar to Bohr's concept of a complete phenomenon.) In the laboratory, the physical systems in question form part of the apparatus used to set up an experiment; but of course unobserved irreversible processes occurring in nature can serve just as well. In quantum mechanics as in classical, human intervention is needed to read the results that have been registered; but is not necessary for such natural quantum-mechanical phenomenon to occur.

perform the ensemble of experiments designed to test whether our answer is valid[w]. But, as we have seen, in general relativity here-and-now and there-and-then cannot be part of the question; they must be part of the answer. So the nature of the questions in a quantum theory that includes general relativity must be of a radically different nature. Again, it is hard to believe that this feature will be lost if we move from general relativity to even more abstract structures, so long as some version or generalization of the dynamic individuation principle is retained.

Now let me finally return to point 1) of my introduction: the basically local nature of general relativity. In all previous physical theories — and in the standard textbook presentations of GR as well — we start we (?????) a preassigned global manifold, on which we impose our fields, dynamical and non-dynamical alike. A singularity of the electromagnetic field does not impose any change in the topology of Minkowski space-time for example. Although the textbooks *say* that is what we do in general relativity, any working relativist knows that is not what we *do* when we set out to solve the field equations. We start with a generic coordinate patch, find a solution to the field equations on that patch, and then try to find the maximal extension of that solution patch into a global manifold subject to certain (still much discussed) criteria (such as, for example, geodesic completeness). So what we would actually do is something analogous to the analytic continuation of a complex function. In that case, starting with a function element, or germ, of a holomorphic function, which we may think of as a local solution $w = f(z)$ to the Cauchy-Riemann equations, one can always construct a Riemann surface on which $w = f(z)$ is a single-valued function[x].

I pointed out this discrepancy between theory and practice some time ago[y]. Since the set of all germs of a holomorphic function has the natural structure of a sheaf, the analogy with analytic continuation suggests that sheaf theory might be the appropriate mathematical tool to handle the problem in general relativity. As far as I know, no one has followed up on this suggestion, and my own recent efforts have been stymied by the circumstance that all treatments of sheaf theory that I know assume an underlying manifold; so, even if one starts with a germ, it is assumed that the manifold that results is known/footnoteOf course, one might start with the large category of sheaves and look for the sheaf that has the global manifold appropriate for the local

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[w] Of course, in practice, and in quantum field theory even in principle, we need regions of space-time rather than points. But this does not affect the argument.
[x] See, for example, the articles "Analytic Functions" and "Riemann Surfaces" in Iyanaga and Kawada 1980[9].
[y] Stachel 1987[15], pp. 205-207.

metric we are investigating; but I guess I am enough of a constructivist in mathematics to feel uneasy about such an approach. See Stachel 2002b[22], Section 1e, for some further comments on this problem..

At any rate, there seems to be something inherently local about general relativity's method of approach to problems; while quantum theory seems to have an inherently global approach to its problems: one needs the sum over *all* paths, for example (NOTA: Although, of course, under certain circumstances, only a few near the classical path make a significant contribution to the total probability amplitude.); and specifying a wave function on just a patch of a manifold is not very helpful. If I am right, this fundamental difference in approach may be one of the deepest reasons for the difficulty of bringing both theories under a common roof. At any rate, one of the problems (NOTA: See the next section for another problem.) I see with all canonical approaches to quantum gravity is that they start from a given global three-dimensional manifold. While this does not fix the four-dimensional global topology, it certainly goes far beyond a patch spatially while demanding much less temporally.

6 The Generalized Permutation Principle

Returning now to the second problem mentioned in the Introduction: as we have seen, in general-relativistic theories, while the points of the manifold are characterized as points of space-time independently of the particular relations in which they stand to each other, they are individuated *entirely* by the relational structure specified by some solution to the generally-covariant field equations. Remarkably enough, the elementary particles are similarly individuated by their position in a relational structure (NOTA: See Stachel 2002a[21] for further discussion of this point.). Each particular kind of elementary particle (e.g., electrons, protons, K-mesons, etc.) may be characterized in a way that is independent of the relational structure in which its exemplars are imbricated: by their mass, spin, charge, half-life, etc.; but a particular elementary particle can only be individuated (to the extent that it can be) by its role in such a structure. (The electrons in an atom, for example, are individuated by the quantum numbers characterizing their place in the electronic structure of that atom.) The reason for this is, of course, the requirement that all relations between N of these particles be invariant under the permutation group acting on these particles (NOTA: This requirement is often referred to as the requirement that elementary particles be either bosons (obey Bose-Einstein statistics) or fermions (obey Fermi-Dirac statistics). While this argument only applies to the treatment of elementary particles in non-relativistic quantum mechanics, it can be generalized to their treatment in (special-)relativistic quantum field theory, by consideration of the appropriate Fock spaces for example.).

But the elementary particles and the points of space-time are the basic building blocks of our current model of the universe. Since they are individuated entirely in terms of the relational structures in which they are embedded, only "higher-level" entities constructed from them can be individuated independently of such relational structures (NOTA: I have discussed the question of the emergence of individuality in somewhat more detail in Stachel 2002c[23].). If individuality has been lost at the level of depth to which we have currently penetrated in our physical theories, it is hard to believe that it will re-emerge if we succeed in penetrating to a deeper level in our understanding of nature. This suggests that we impose the following *generalized permutation principle* as a requirement on any candidate for a future (more) fundamental theory:

Whatever the nature of the basic elements out of which it is constructed, the theory should be invariant under *all* permutations of these basic elements.

It is by no mean certain that the space-time manifold itself will form a basic element of such deeper structure(s). It may turn out that space-time itself is built out of quantized elements as the loop quantum gravity program suggests. At an even deeper level, it may turn out that space-time itself is a construct, built out of some radically different units. Again, it is hard to believe that these units, whatever their nature, will regain an individuality already lost at the classical space-time level. If my argument is correct, then the generalized permutation principle should be applicable to them.

How do current candidates for a theory of quantum gravity fare when examined in the light of this principle? The quantum gravity community splits rather clearly into two sub-communities: those who approach the problem with a background (primarily) in general relativity, and those who approach the problem with a background (primarily) in quantum field theory. The approach most used by the first group (at least until very recently) has been canonical quantization; the approach favored by the second community (at least until recently) has been string theory. I shall look briefly at these two approaches.

Canonical quantization, in all its variants, is based on the imposition of additional geometrical structures on the four-dimensional space-time manifold: a foliation and a fibration that break up space-time into a three-dimensional space and a one-dimensional time. Restricting the diffeomorphism group to the subgroup that preserves these structures clearly violates the generalized covariance principle. I believe that this violation is responsible for many of the problems that confront this approach, notably the notorious "problem of time". I think most general relativists would agree that in principle it would be better to have an approach that does not violate the principle, and much recent work in the field of loop quantum gravity and spin networks has been devoted to attempts to find a four-dimensional formulation.

Turning to string theory, perturbative string theory fails the test, since the background space-time (of no matter how many dimensions) is only invariant under a finite-parameter Lie subgroup of the group of all possible diffeomorphisms of its elements. Since many string theorists, coming to the field with a special-relativistic quantum field theory background, intially found it difficult to accept this criticism, I find it encouraging that this point now seems to be widely acknowledged in the string community. I quote from two recent review articles. Speaking of the original string theory Michael Green notes:

> This description of string theory is wedded to a semiclassical perturbative formulation in which the string is viewed as a particle moving through a fixed background geometry. ... Although the series of superstring diagrams has an elegant description in terms of two-dimensional surfaces embedded in spacetime, this is only the perturbative approximation to some underlying structure that must include a description of the quantum geometry of the target space as well as the strings propagating through it (Green 1999[8], p. A78). ... A conceptually complete theory of quantum gravity cannot be based on a background dependent perturbation theory. ... In ... a complete formulation the notion of string-like particles would arise only as an approximation, as would the whole notion of classical spacetime (ibid., p. A 86).

Speaking of the more recent development of M theory, Green says:

> An even worse problem with the present formulation of the matrix model is that the formalism is manifestly background dependent. This may be adequate for understanding M theory in specific backgrounds but is obviously not the fundamental way of describing quantum gravity (ibid., p. A 96).

And in a review of matrix theory, Thomas Banks comments (Banks 1998[2]):

> String theorists have long fantasized about a beautiful new physical principle which will replace Einstein's marriage of Riemannian geometry and gravitation. Matrix theory most emphatically does not provide us with such a principle. Gravity and geometry emerge in a rather awkward fashion, if at all. Surely this is the major defect of the current formulation, and we need to make a further conceptual step in order to overcome it (pp. 181-182).

It is my hope that emphasis on the importance of the principle of dynamic individuation of the fundamental entities, with its corollary requirement of invariance of the theory under the entire permutation group acting on these entities, constitutes a small contribution to taking that further conceptual step.

References

1. Ashtekar, Corichi, and Pierri 1999: Abhay Ashtekar, Alejandro Corichi and Monica Pierri, "Geometry in Color Perception", in *Black Holes, Gravitational Radiation and the Universe*, Bala Iyer and Biplap Bhawal, eds. (Kluwer Academic, Dodrecht/Boston/London 1999), pp. 535-549.
2. Banks 1998: Thomas Banks, "Matrix Theory", *Nuclear Physics B (Proceedings Supplement)* **67** (1998): 180-224.
3. Bronstein 1933: Matvey Petrovich Bronstein, "K voprosy o vozmozhnoy teorii mira kak tselogo" ["On the Question of a Possible Theory of the World as a Whole"], *Uspekhi Astronomicheskikh Nauk. Sbornik* **3**: 3-30.
4. Earman and Norton 1987: John Earman and John Norton, "What Price Spacetime Substantivalism? The Hole Story", *British Journal for the Philosophy of Science* **38**: 515-525.
5. Ehlers 1973a: Jürgen Ehlers, "Survey of General Relativity Theory", in *Relativity, Astrophysics and Cosmology*, Werner Israel, ed. (Reidel, Dordrecht 1973), pp. 1-125.
6. Ehlers 1973b: Jürgen Ehlers, "The nature and structure of spacetime", in *The Physicists Concept of Nature*, Jagdish Mehra, ed. (Reidel, Dordrecht/Boston 1973).
7. Einstein-Besso 1972: *Albert Einstein and Michele Besso, Correspondance 1903-1955*, Pierre Speziali, ed. (Hermann, Paris 1972).
8. Green 1999: Michael B. Green, "Superstrings, M theory and quantum gravity", *Classical and Quantum Gravity* **16** (1999): A77-A100.
9. Iyanaga and Kawada 1980: Shkichi Iyanaga and Yukiyosi Kawada, eds., *Encyclopedic Dictionary of Mathematics*, 2 vols, paperback ed. (MIT Press, Cambridge, MA/London 1980).
10. Klein 1872: Felix Klein, *Vergleichende Betrachtungen über neuere geometrische Forschungen* (A . Dchert, Erlangen 1872), rev. version in *Mathematische Annalen* **43**: 63-100 (1893).
11. Rovelli 1998: Carlo Rovelli, "Strings, loops and others: a critical survey of the present approaches to quantum gravity", in *Gravitation and Relativity at the Turn of the Millenium*, N. Dadhich and J. Narlikar, eds. (Inter-University Centre for Astronomy and Astrophysics, Pune 1998), pp. 281-

331.
12. Shafarevich 1997: I.R. Shafarevich, *Basic Notions of Algebra* (Springer, Berlin etc. 1997), Section 1, “”What is Algebra”, pp. 6-11, Section 17, “Group Representations”, pp 160-162.
13. Sorkin 1997: Rafael D. Sorkin, “Forks in the Road, on the Way to Quantum Gravity”, *International Journal of Theoretical Physics* **36**: 2759-2781.
14. Stachel 1983: John Jay Stachel, “Special Relativity from Measuring Rods”, in *Physics, Philosophy, and Psichoanalysis/Essays in Honor of Adolf Grünbaum*, Robert S. Cohen and Larry Laudan, eds. (Reidel, Dordrecht/Boston/Lancaster 1983), pp. 255-272.
15. Stachel 1987: John Jay Stachel, “How Einstein discovered general relativity: a historical tale with some contemporary morals”, in *General Relativity and Gravitation/Proceedings of the* 11^{th} *International Conference on General Relativity and Gravitation*, Malcolm A. H. MacCallum, ed. (Cambridge University Press, Cambridge 1987), pp. 200-208. Reprinted in John Stachel, *Einstein from ‘B’ to ‘Z’* (Birkhäuser, Boston/Basel/Berlin 2002), pp. 293-299.
16. Stachel 1989: John Jay Stachel, “Einstein’s Search for General Covariance, 1912-1925”, in *Einstein and the History of General Relativity* (*(Einstein Studies*, vol. 1), Don Howard and John Stachel, eds. (Birkhäuser, Boston/Basel/Berlin 1989), pp. 63-100. Reprinted in John Stachel, *Einstein from ‘B’ to ‘Z’* (Birkhäuser, Boston/Basel/Berlin 2002), pp. 301-337.
17. Stachel 1993a: John Jay Stachel, “The Meaning of General Covariance: The Hole Story”, in *Philosophical Problems of the Internal and External World*, John Earman et al, eds. (University of Pittsburgh Press/Universitätsverlag Konstanz 1993), pp. 129-160.
18. Stachel 1993b: John Jay Stachel, “The Other Einstein: Einstein Contra Field Theory”, in *Science in Context 6* (1993), pp. 275-290. Reprinted in Reprinted in John Stachel, *Einstein from ‘B’ to ‘Z’* (Birkhäuser, Boston/Basel/Berlin 2002), pp. 141-154.
19. Stachel 1999a: John Jay Stachel, “Introduction to ‘Quantum field theory and space-time”’, in *Conceptual Foundations of Quantum Field Theory*, Tian Yu Cao, ed. (Cambridge University Press, Cambridge 1999), pp. 166-175.
20. Stachel 1999b: John Jay Stachel, “Espace-Temps”, in *Dictionnaire d’histoire et philosophie des sciences*, Dominique Lecourt, ed. (Presses Universitaires de France, Paris 1999), pp. 376-379. “Space-Time”, a revised and expanded version prepared for the English edition of this

dictionary, is availabe from the author.
21. Stachel 2002a: John Jay Stachel, "'The Relations between Things' versus 'The Things between Relations': The Deeper Meaning of the Hole Argument", in *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*, David Malament, ed. (Open Court, Chicago and LaSalle 2002), pp. 231-266.
22. Stachel 2002b: John Jay Stachel, "Fibered Manifolds, Geometric Objects, Structured Sets, G-Spaces and All That: The Hole Story from Space-Time to Elementary Particles", preprint available from the author.
23. Stachel 2002c: John Jay Stachel, "Structure and Individuality", preprint available from the author.
24. Weyl 1946: Hermann Weyl, *The Classical Groups/Their Invariants and Representations*, 2^{nd} ed. (Princeton University Press, 1946), Chapter I, Section 4, "Groups, Klein's Erlanger Program. Quantities", pp. 13-23.
25. Weyl 1949: Hermann Weyl, *Philosophy of Mathematics and Natural Science*, rev. and augmented English ed., Olaf Helmer, transl. (Princeton University Press 1949), "Part I. Mathematics" (pp. 3-91), especially Chap. III, section 13, "The Problem of Relativity" (pp. 71-78).