Analog Duality

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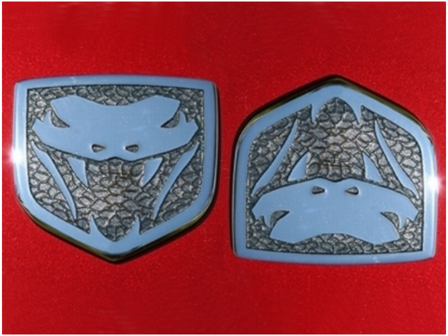
Dualities

- A duality, in the broadest sense, identifies two theories with each other.
- A duality is especially interesting if the two theories are very different.
- Long history. Received much attention recently because of dualities that have been discovered in string theory.
- Dualities are a game-changer for the search of the 'fundamental' theory because they question what is emergent and what is fundamental.
- Also of practical use as calculation tools.

Unification vs Duality

A duality is a type of unification, but it does not work by combining two theories into a larger whole. Instead it unifies by showing that two theories are actually identical.

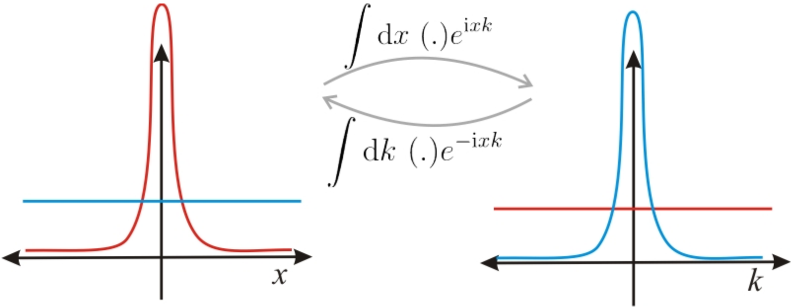
Unified Snake-Duck



Snake-Duck Duality

Particle-Wave Duality

- Not the type of theory-duality that we are interested in, but instructive nevertheless.
- Wave-functions can be more particle-like or be more wave-like.
- Fourier transform: Any particle (localized delta) is composed of infinitely many waves, and vice versa.
- This relation is strongly **non-local**.
- A wavepacket of width Δx in position space has a width $1/\Delta x$ in momentum space.



Self-Duality in Electrodynamics

Free Electrodynamics in 3+1 dimensions. $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\kappa} F_{\alpha\kappa}/2$

$$\begin{aligned} Z &= \int \mathcal{D}A e^{-\mathrm{i}F^2/(4e^2)} = \int \mathcal{D}F \prod_x \delta(\partial_\nu \tilde{F}^{\mu\nu}) e^{-\mathrm{i}F^2/(4e^2)} \\ &= \int \mathcal{D}F\mathcal{D}V \exp \mathrm{i} \left(\int d^4x V_\nu \partial_\mu \tilde{F}^{\mu\nu} - F^2/4e^2 \right) \end{aligned}$$

Eom: $\tilde{F}_{\mu\nu} \sim \partial_\nu V_\mu - \partial_\mu V_\nu$. Partial integration, then integrate over F

$$Z = \int \mathcal{D}V e^{-\mathrm{i}e^2/(16\pi^2)\tilde{F}^2}$$

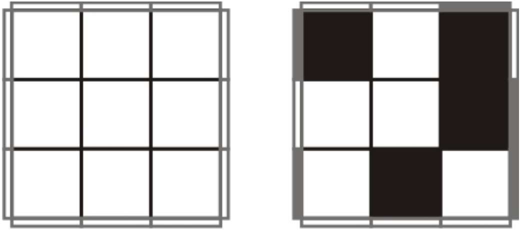
Swaps electric fields with magnetic fields. The coupling $1/e^2$ becomes e^2. This is a free theory, so the coupling could be absorbed in the fields, but one can see here how a large coupling constant can correspond to a small one in the dual theory!

Gauge-gravity duality

- Identifies classical gravity in AdS space with a gauge theory on the (flat) **boundary** of that space.
- It is a strong-weak duality: If the gauge-theory is strongly coupled, the gravity side is weakly coupled.
- Best understood case $\mathcal{N}=4$ SYM, $\mathrm{AdS}_5 \times \mathrm{S}^5$. Some other cases with less supersymmetry.
- Duality has not been proved but at least for these cases there is little doubt.
- **AdS space is not globally hyperbolic**
- Believed to solve the black hole information loss problem.

Holography

- The gauge-gravity duality is said to be 'holographic' because the physics of the whole AdS space is encoded on the boundary.
- This is surprising because it restricts the number of degrees of freedom.



How many dimensions do we ‘really’ live in?

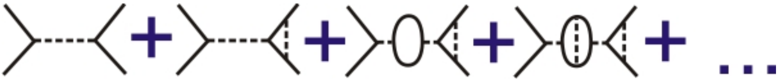
- Depends on how you define dimension.
- Physically meaningul: Spectral dimension. Is defined by a diffusion process whose return time depends on the number of dimensions.
- What we mean with ‘dimension’ depends on what a ‘particle’ is.
- So: ‘we’ (weakly coupled) live in 3+1 dimensions (plus possible compactified dimensions that are only accessible at high energies), and not on a lower dimensional boundary.
- A strongly coupled system in our lab (3+1) could reasonably be said to actually live in higher dimensions (4+1).

What is fundamental?

- You can take either side of the duality as ‘fundamental’, then the other side is ‘emergent’.
- Thus the notion of ‘fundamental’ fields, interactions or symmetries is relative.
- What is fundamental is not the realization of the theory, but the underlying principles.
- Path integral, quantization, consistency conditions like a stable vacuum, well-defined initial value problems.
- String theorists believe string theory to be the *only* consistent theory that does all that, plus the relevant limits to GR and the SM.
- (That might be correct. Otoh, we don’t know that these fundamental assumptions are actually correct.)

What are dualitites good for?

- Most of particle physics relies on perturbation theory. This means we have free particles (possibly over a background field) with a small interaction probability.
- Expansion of S-matrix in Feynman diagrams

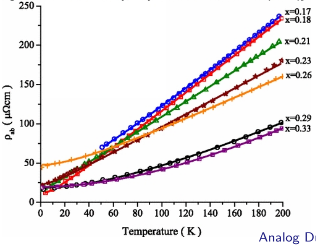


- This only works if the particles interact weakly. Otherwise the corrections don't get smaller.
- If the corrections don't get smaller, one needs **non-pertubative** methods. These are difficult.
- A weak-strong duality can convert a non-perturbative problem into a perturbative one by changing the field content.

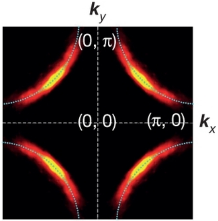
Strongly Coupled Systems

- Strongly coupled systems cannot be treated with perturbation theory.
- Prominent examples are the quark gluon plasma (or nuclear matter at low energy generally) and strange metals (including high-temperature superconductors).
- Strange metal are strange because they have an unusual scaling of resistivity with temperature (linear instead of quadratic, keeps on growing) and don't seem to have quasi-particles. This indicates BCS theory doesn't work.
- Using the gauge-gravity duality is one way to address this problem.
- (Of course not everybody agrees and AdS/CMT isn't the only approach on the market.)

[Image: Anderson & Casey, Phys. Rev. B 80, 094508 (2009)]



[Image:Keimer et al, Nature 518, 179 (2015)]



The Short Story

Gauge-gravity duality: Short Story

- AdS/CFT discovered through string-gauge duality.
- It identifies a II B string theory in AdS space with a gauge theory on the boundary of that space.
- In the large N limit, the string theory becomes classical. At large λ, string effects are supressed $\longrightarrow$ There is some limit in which we have a duality between classical gravity in AdS and a strongly coupled system on the boundary of AdS.
- Point of view here: It is a well-founded motivation to use gravitational systems as models for strongly coupled systems.
- In particular: Strange metals near quantum criticality, for which conformal invariance should be a good approximation.

Analog Gravity: Short Story

- Small perturbations travelling in (or on) fluids fulfill an equation of motion analytically identical to the wave-equation in a curved space.
- One can assign an effective metric to the fluid background, which is a function of the fluid's variables $(\rho, p, \vec{v})$.
- Best known example: Unruh's dumb hole.
- Can be understood as a weak-weak duality for perturbation.
- The background's equations of motion will not generally reproduce the field equations. (This does not mean they cannot!)

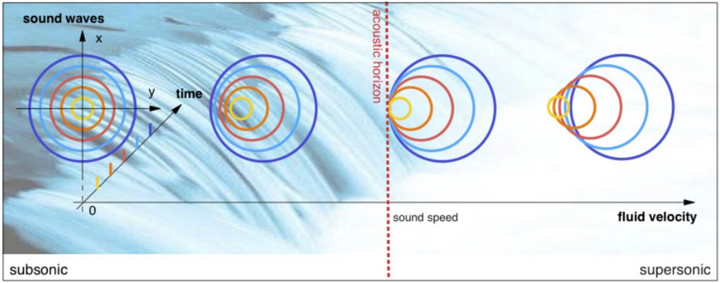
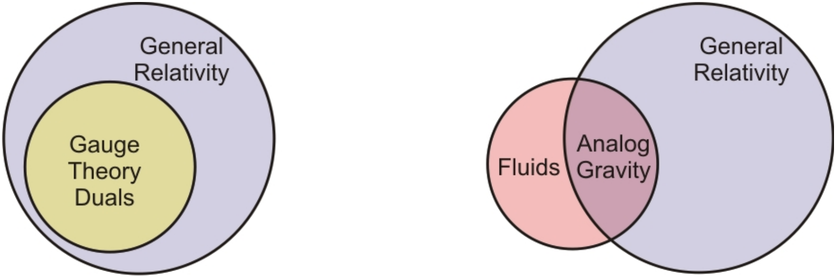


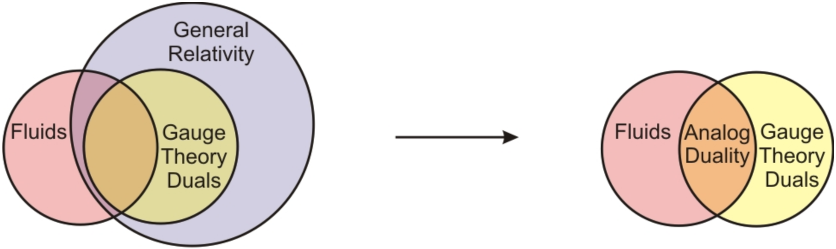
Image Credits: Barcelo, Liberati & Visser

Analog Duality: Short Story



- Some solutions to Einstein's field equations describe strongly coupled condensed matter systems via the gauge-gravity duality.
- Some metrics can be obtained as effective metrics in weakly coupled condensed matter systems.

Analog Duality: Short Story



- Show that some of the AdS metrics dual to strongly coupled systems can also be analog gravity systems.
- Then this results in a strong-weak duality among condensed matter systems.

The Long Story

AdS/CMT: Long Story

- The gravitational system to model a holographic superconductor is coupled to a U(1) charged, massive scalar field

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{d+1}x\sqrt{-g}\left(\mathcal{R} - \Lambda - \frac{1}{4}F^2 - V(|\psi|) - |\partial\Psi - \mathrm{i}qA\Psi|^2\right) \quad .$$

- In the 'probe limit' (no backreaction) this is qft in curved space with a metric of a charged, planar, black hole

$$\begin{aligned}
\mathrm{d}s^2 &= -\frac{L^2}{z^2}\gamma(z)\mathrm{d}t^2 + \frac{L^2}{z^2}\gamma(z)^{-1}\mathrm{d}z^2 + \frac{L^2}{z^2}\sum_{i=1}^{d-1}\mathrm{d}x^i\mathrm{d}x^i \ . \\
\gamma(z) &= 1 - (1+\alpha^2)\left(\frac{z}{z_0}\right)^d + \alpha^2\left(\frac{z}{z_1}\right)^{2(d-1)} \ , \ A_t \sim 1 - \left(\frac{z}{z_0}\right)^{d-2}
\end{aligned}$$

- These planar black holes can only exist in asymptotic AdS.
- Of course grossly simplified. Don't expect quantitative results.

Holographic superconductors

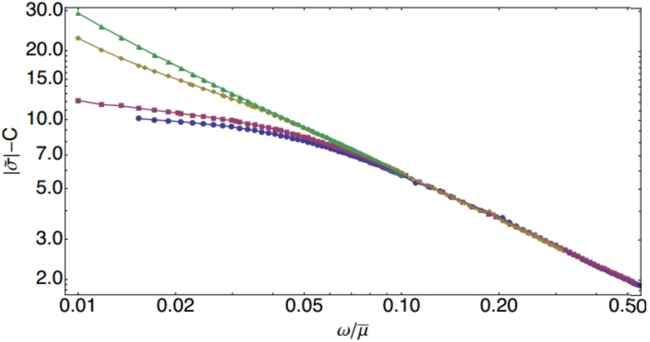
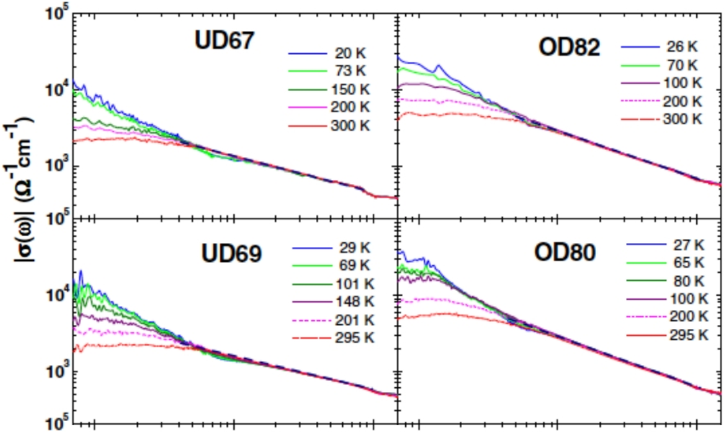


Fig 8, Horowitz & Santos, arXiv:1302.6586 [hep-th]
Frequency-dependence of conductivity scales with $\omega^{-2/3}$ above and below transition temperature.

Holographic superconductors



Timusk & Gu, arXiv:cond-mat/0607653
Measured frequency-dependence of conductivity above and below transition temperature.

Analog Gravity: Long Story

- The effective analog metric of a (non-relativistic) fluid takes the form

$$g_{\mu\nu}(t,\vec{x}) \quad \propto \quad \left(\frac{\rho}{c}\right)^{\frac{2}{n-1}} \left(\begin{array}{cc} -(c^2 - v^2) & -v^j \\ -v^i & \delta_{ij} \end{array} \right) .$$

- Note that the scaling depends on the number of dimensions!
- Procedure:

  1. Rewrite metric into the above form. This will not in general be possible.

  2. Read off fluid's degrees of freedom.

  3. Check that these degrees of freedom fulfil the fluid's equation of motions. (Euler equation and continuity equation, or relativistic versions respectively.) Again, this will not in general be the case.

The analog Schwarzschild black hole

- Schwarzschild metric in Painlevé-Gullstrand coordinates coordinates

$$\mathrm{d}s^2 = -\gamma \mathrm{d}t'^2 + \sqrt{\frac{2MG}{r}} \mathrm{d}t' \mathrm{d}r + r^2 + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \right) \ .$$

- Can read off $c = \rho$, $c = 1$, and

$$\rho v = \sqrt{\frac{2MG}{r}} \ .$$

- Does **not** automatically fulfill the continuity equation $\partial_r(\rho v) = 0$!
- Introduct conformal pre-factor, and it does.
- Changes overall scaling of propagating modes, but this can be adjusted for analytically.
- Conformal factor unsatisfactory for duality idea.

Empty AdS

- Is trivial because conformally flat.

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left(-dt^2 + dz^2 + \sum_{i=1}^{d-1} \mathrm{d}x^i \mathrm{d}x^i\right) \quad .$$

- Is a fluid background with $\rho \sim 1/z^2, \vec{v} = 0$ and $c = 1$...
- Rescale $t \to t\kappa$, then $c = \kappa$

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left(-\kappa^2 dt^2 + dz^2 + \sum_{i=1}^{d-1} \mathrm{d}x^i \mathrm{d}x^i\right) \quad .$$

- Works, but rather boring.

The AdS Planar black hole

- Convert by same method as Schwarzschild black hole. Gives

$$\mathrm{d}s^2 = -\frac{L^2}{z^2}\left(1 - \frac{z^d}{z_0^d}\right)\kappa^2 \mathrm{d}t'^2 - \frac{L^2}{z^2}\left(\frac{z}{z_0}\right)^{d/2} \kappa \mathrm{d}t' \mathrm{d}z + \frac{L^2}{z^2}\sum_{i=1}^{d-1} \mathrm{d}x^i \mathrm{d}x^i \ .$$

- Read off $c = \kappa$, $\rho \sim 1/z^2$, $v_z = v = \kappa(z/z_0)^{(d/2)}$.
- Continuity equation

$$\partial_z \rho v \propto \partial_z z^{d/2-2} = 0$$

- The AdS planar black hole (dual to a 3+1 dimensional strongly coupled system) **automatically** fulfills the fluid equation of motion in $4+1$ spatial dimensions
- The analog gravity system generates a 3+1 dimensional slice of the AdS space.
- This only works in the right number of dimensions and it only works in asymptotic AdS.

The Charged AdS Planar black hole

- Convert by same method as Schwarzschild black hole.

$$\mathrm{d}s^2 = -\frac{L^2}{z^2}\gamma(z)\kappa^2\mathrm{d}t'^2 - \frac{L^2}{z^2}\sqrt{\gamma(z)-1}\kappa\mathrm{d}t'\mathrm{d}z + \frac{L^2}{z^2}\sum_{i=1}^{d-1}\mathrm{d}x^i\mathrm{d}x^i \ ,$$

where now

$$\gamma(z) = 1-(1+\alpha^2)\left(\frac{z}{z_0}\right)^d + \alpha^2\left(\frac{z}{z_1}\right)^{2(d-1)}$$

- Seems to give $\rho \sim 1/z^2$ and $v^2 \sim \gamma(z)-1$, so that $\partial_z(\rho v) \neq 0$.
- BUT: Cannot read off physical quantities as previously because metric now contains gauge-covariant derivative.
- Way to go: Use Lagrangian approach for fluid. Use minimal coupling. Rederive effective metric. Take non-relativistic limit. Then identify correct velocity.

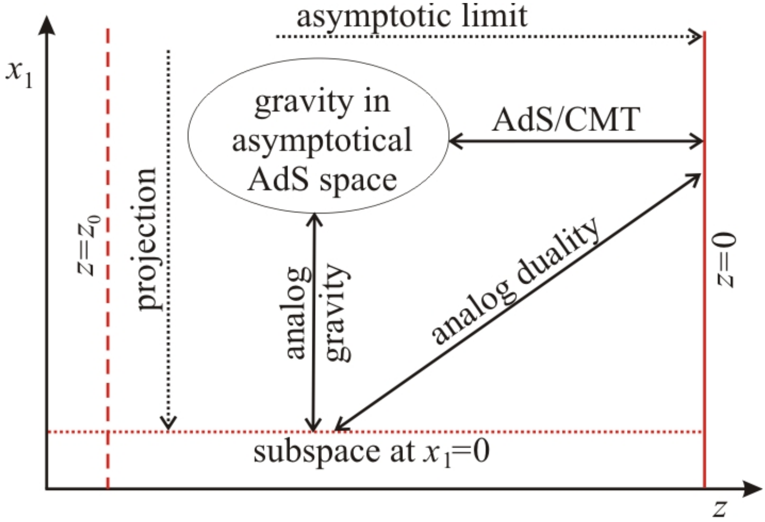
The Charged AdS Planar black hole

- From non-perturbed equations of motion and field A_ν get current j_ν.
- From A_ν and j^ν get coupling term $A_\nu j^\nu$.
- It turns out that the real velocity

$$v \sim \sqrt{\frac{\gamma - 1}{1 - j_\nu A^\nu}} \quad \text{with} \quad j_\nu A^\nu \sim z^{d-4} \left(1 - \left(\frac{z}{z_0}\right)^{d-2}\right)$$

- This means for d=4: $\gamma - 1 \sim z^4(1 - j \cdot A)$, $v^2 \sim z^4$, $v \sim z^2$, and $\partial_z \rho v = 0$.
- **The charged AdS planar black hole still corresponds to another fluid that automatically fulfills the equations of motion!**
- Putting in all the constants relates the AdS chemical potential with the charge density of the fluid and the AdS temperature with the speed of sound.
- Is this a coincidence?

Sketch of idea



What is it good for?

- New method, opens new options to solve existing problems (different set of equations).
- Both systems can be realized in the laboratory, so they can be compared directly by making measurements rather than numerical simulation, which can increase the number of case that can be looked at.
- Since the duality relies on the AdS/CMT duality, the experimental test serves to implicitly **experimentally test the AdS/CMT duality**. (Compares data to data, not calculation to data.)

Take home message

This could be first evidence for a new duality between strongly and weakly coupled condensed matter systems.

Fineprint

- This does not take into account backreaction. This is only field theory in curved space.
- This is only the non-relativistic limit. There should be a relativistic completion.
- This is not the analog metric for a quantum field but for a classical field (the identification of dof looks different)
- There must be some dof getting lost because of the projection.
- The Euler-equation does not give an additional constraint.
- The potential on the AdS side isn't fixed by the general Lagrangian approach. It can be chosen so that the mass of the scalar particle is constant.