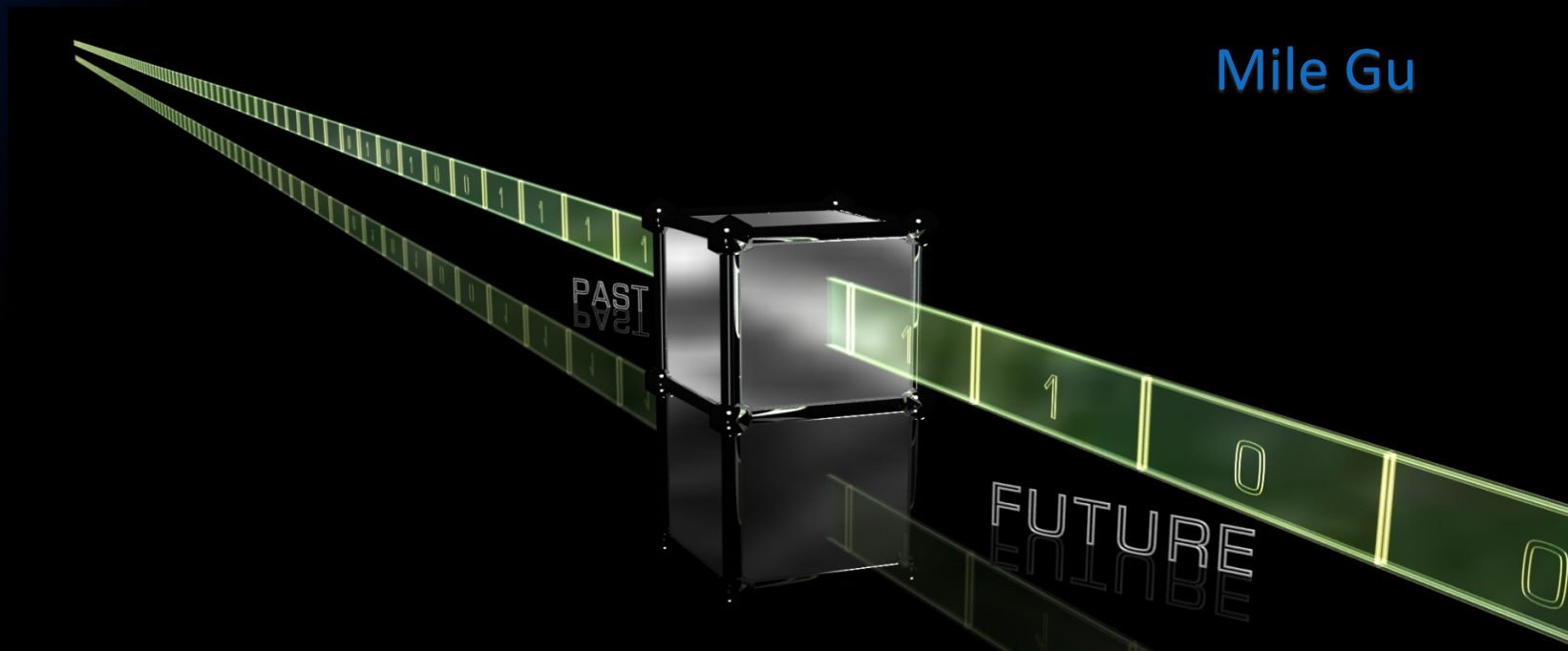
Quantum Simplicity

Can Quantum Theory Better Isolate the Causes of Natural Phenomena?

Mile Gu



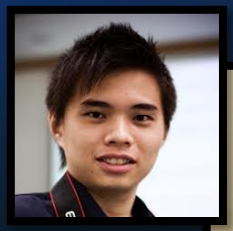
John Templeton Foundation

清华大学量子信息中心
Center for Quantum Information, Tsinghua University

CQT Centre for Quantum Technologies
National University of Singapore

26/4/2015
New Directions in the Foundations of Physics

COLLABORATORS



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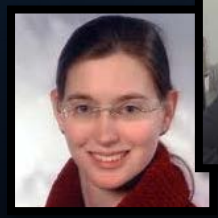

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Alex Monras

University of BRISTOL

Karoline Wiesner

Narrative

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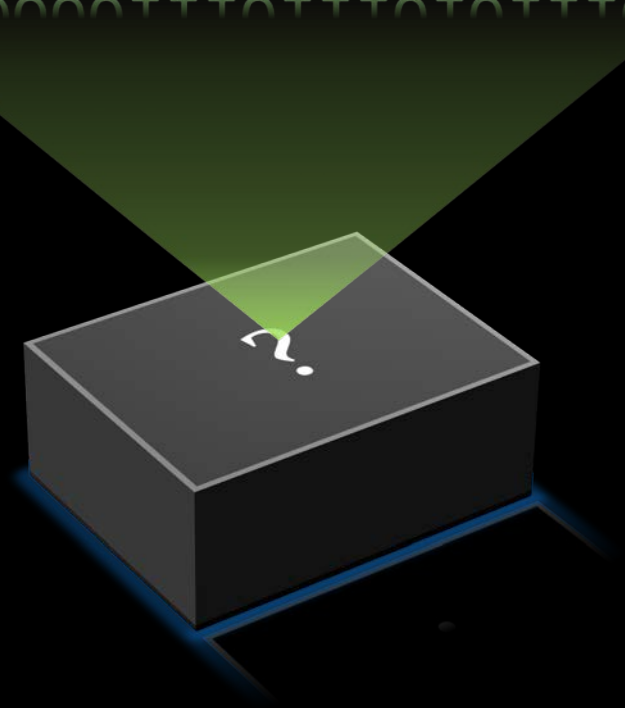
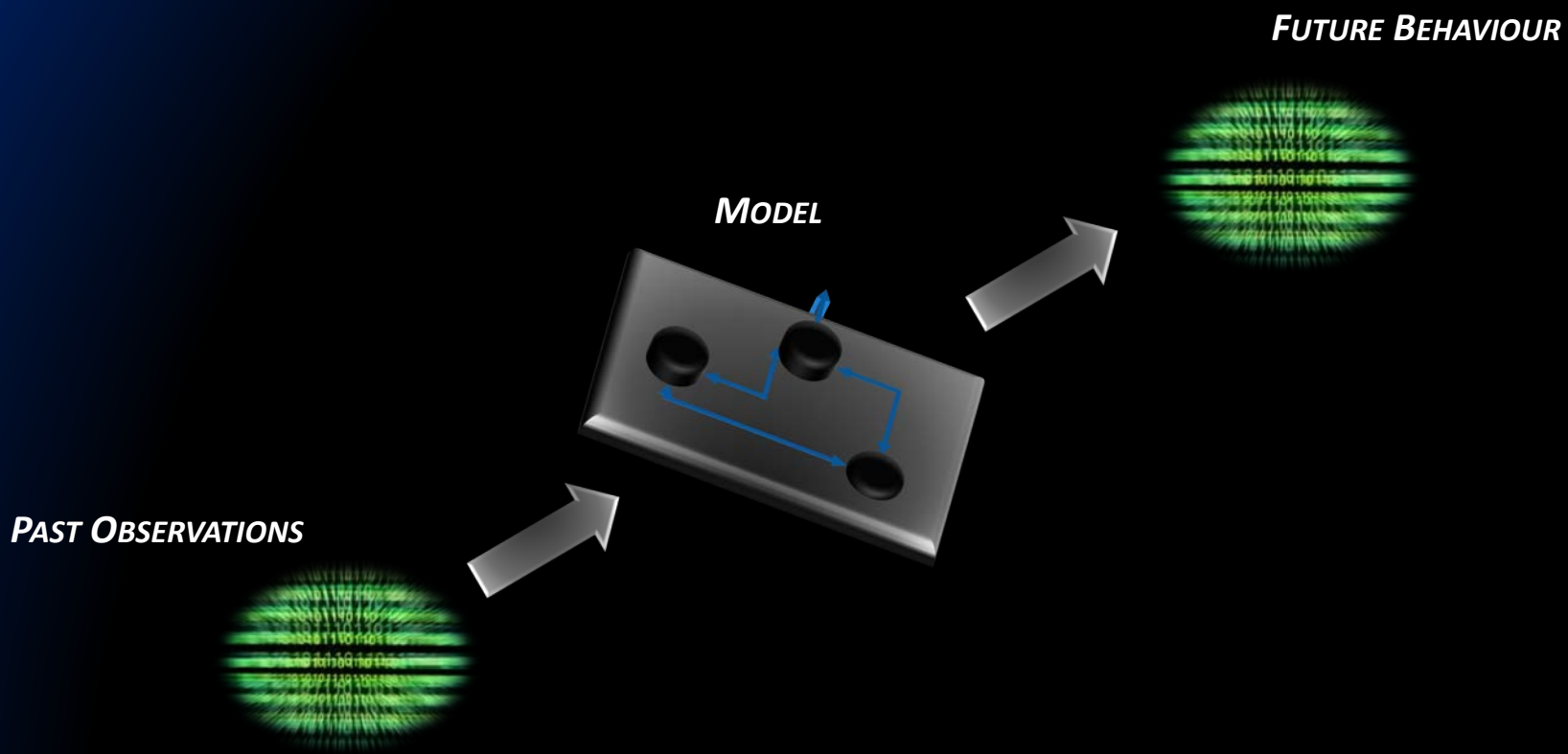
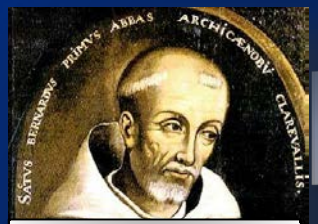


NARRATIVE



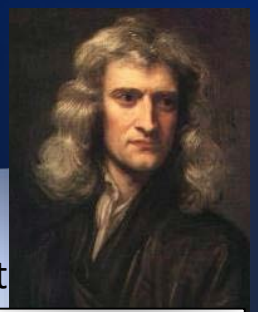
Occam's Razor



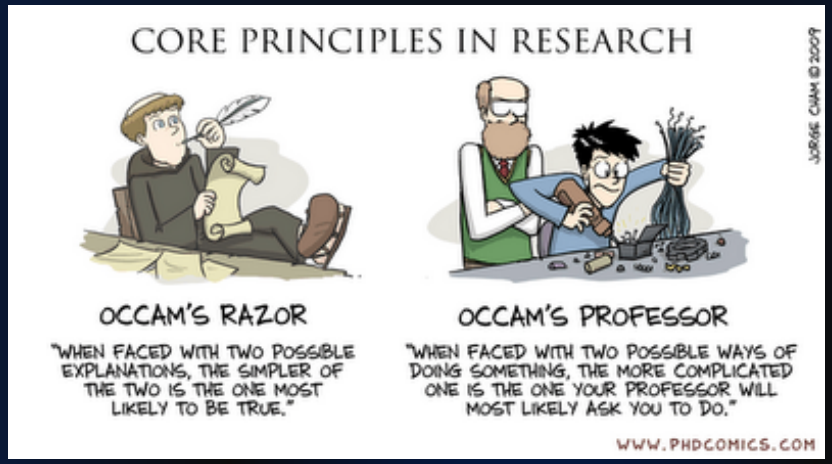
William of Ockam

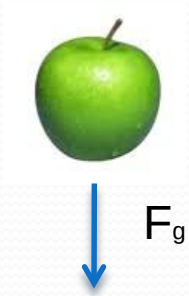
"Plurality is not to be posited without necessity."

"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."



Isaac Newton





Initial Position

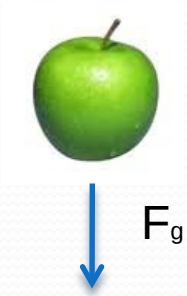
Initial Velocity

Model

$$\frac{d^2y}{dt^2} = -g$$

Observed Behavior

$$y(t)$$



Initial Position

Initial Velocity

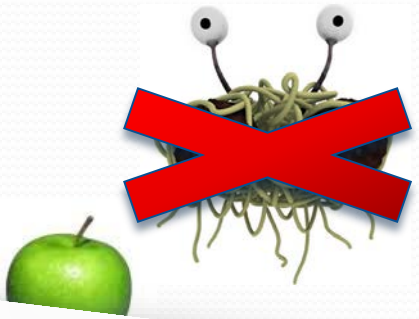
Entire Past History

Model

$$\frac{d^2y}{dt^2} = -g$$

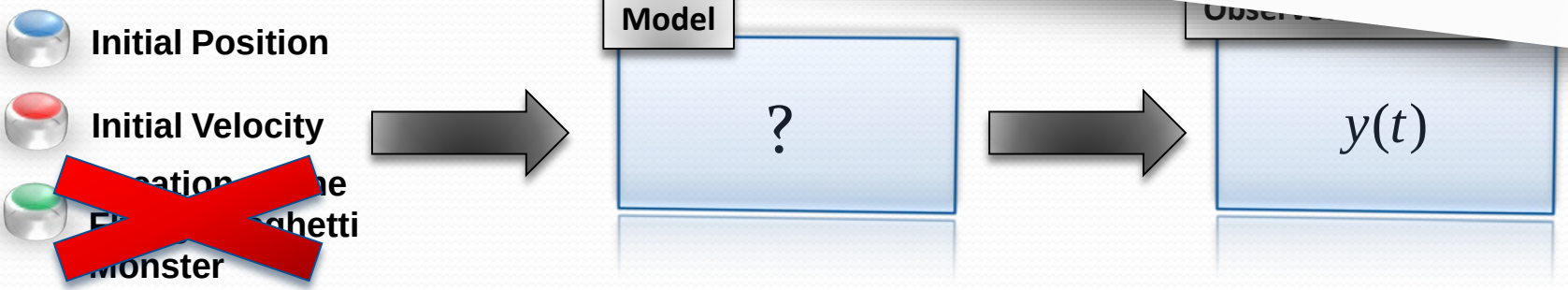
Observed Behavior

$$y(t)$$

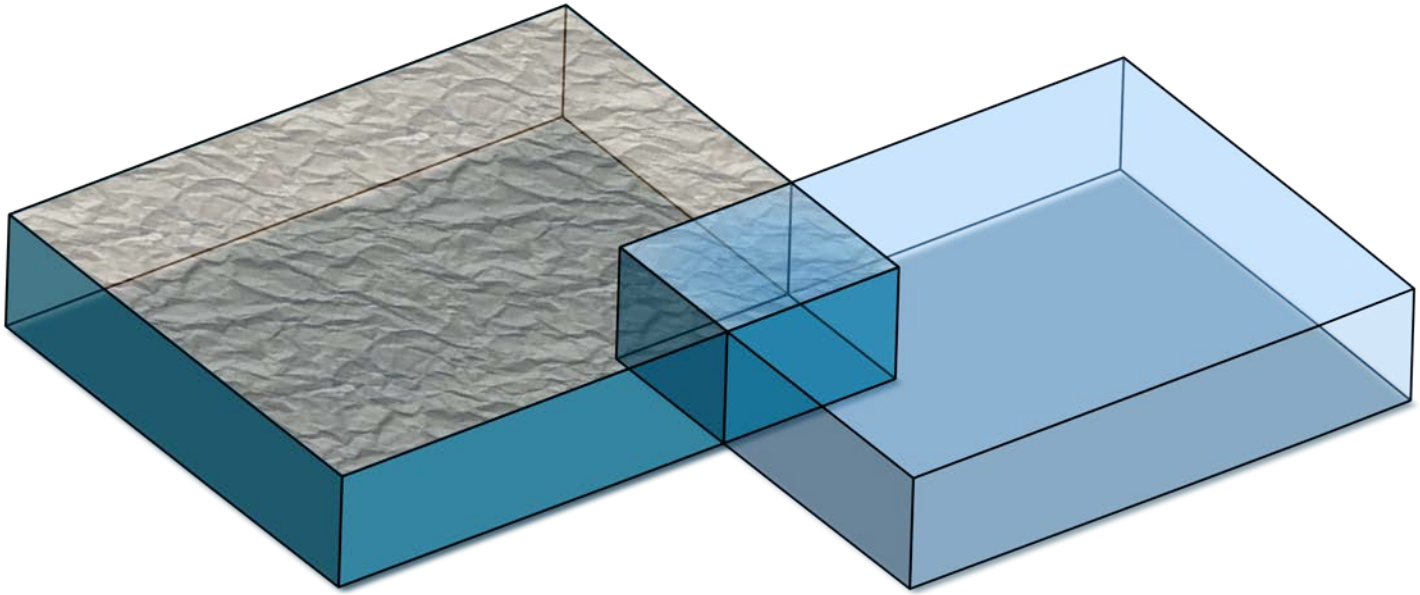


IN ADDITION, there are a number of lesser ways in which the manuscript should be improved;

1. The authors should show due respect by referring to "The Flying Spaghetti Monster" rather than "a flying spaghetti monster".

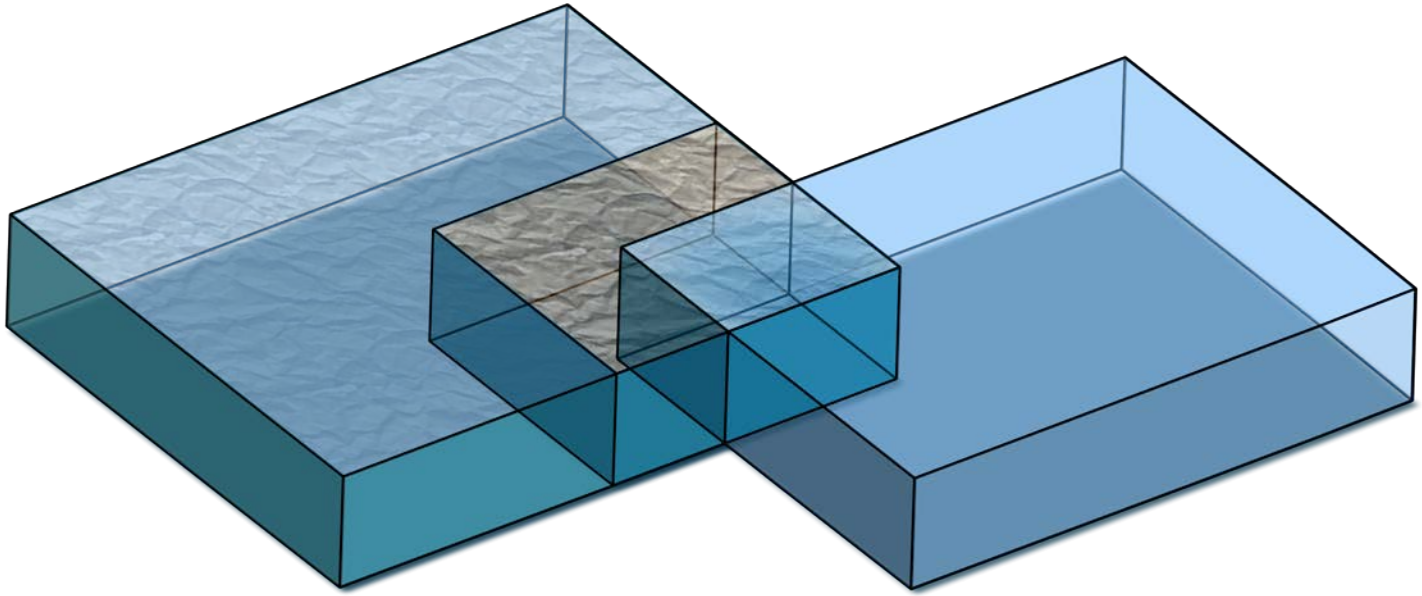


Without any understanding, every possible past is a potential cause of future events.

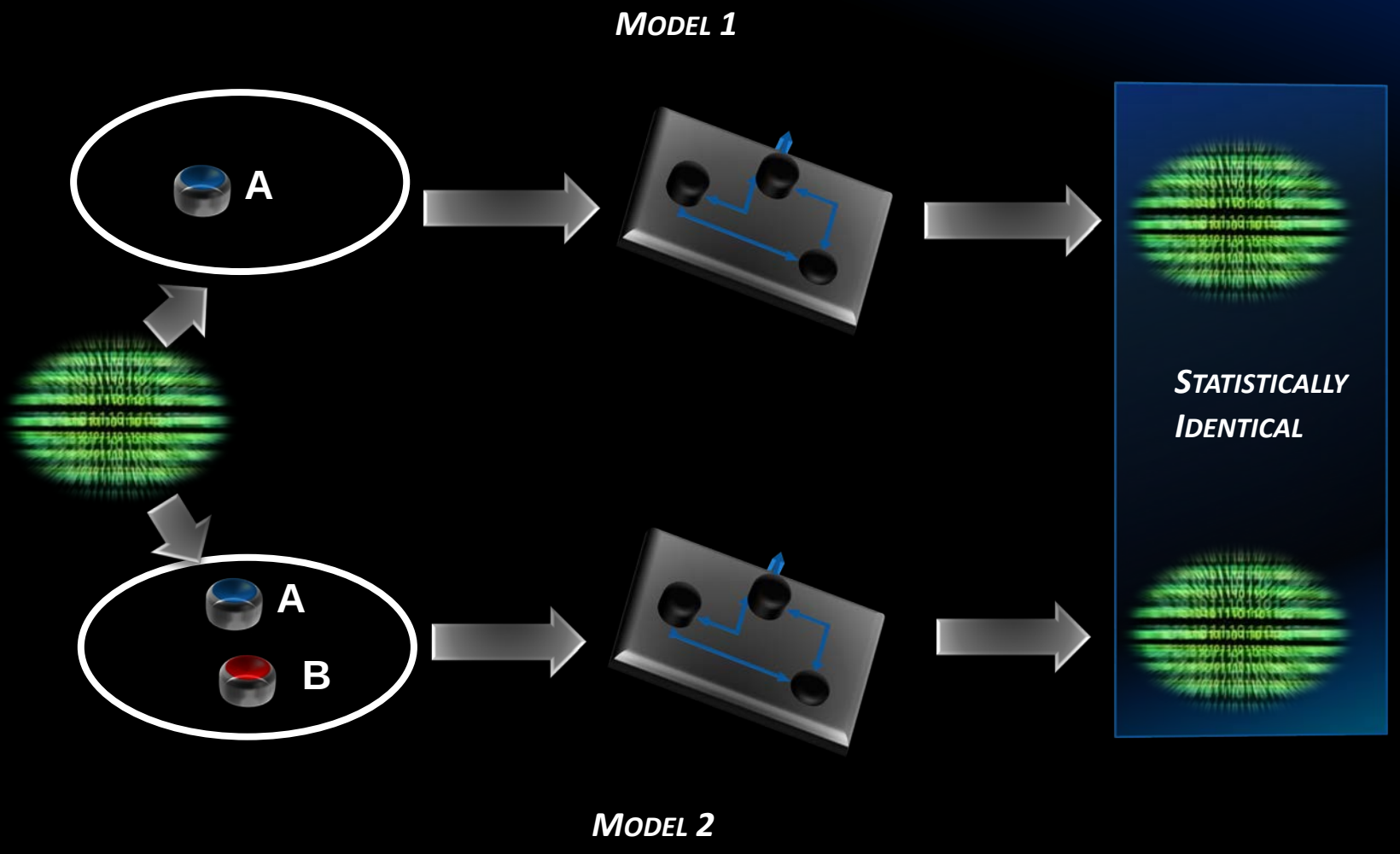


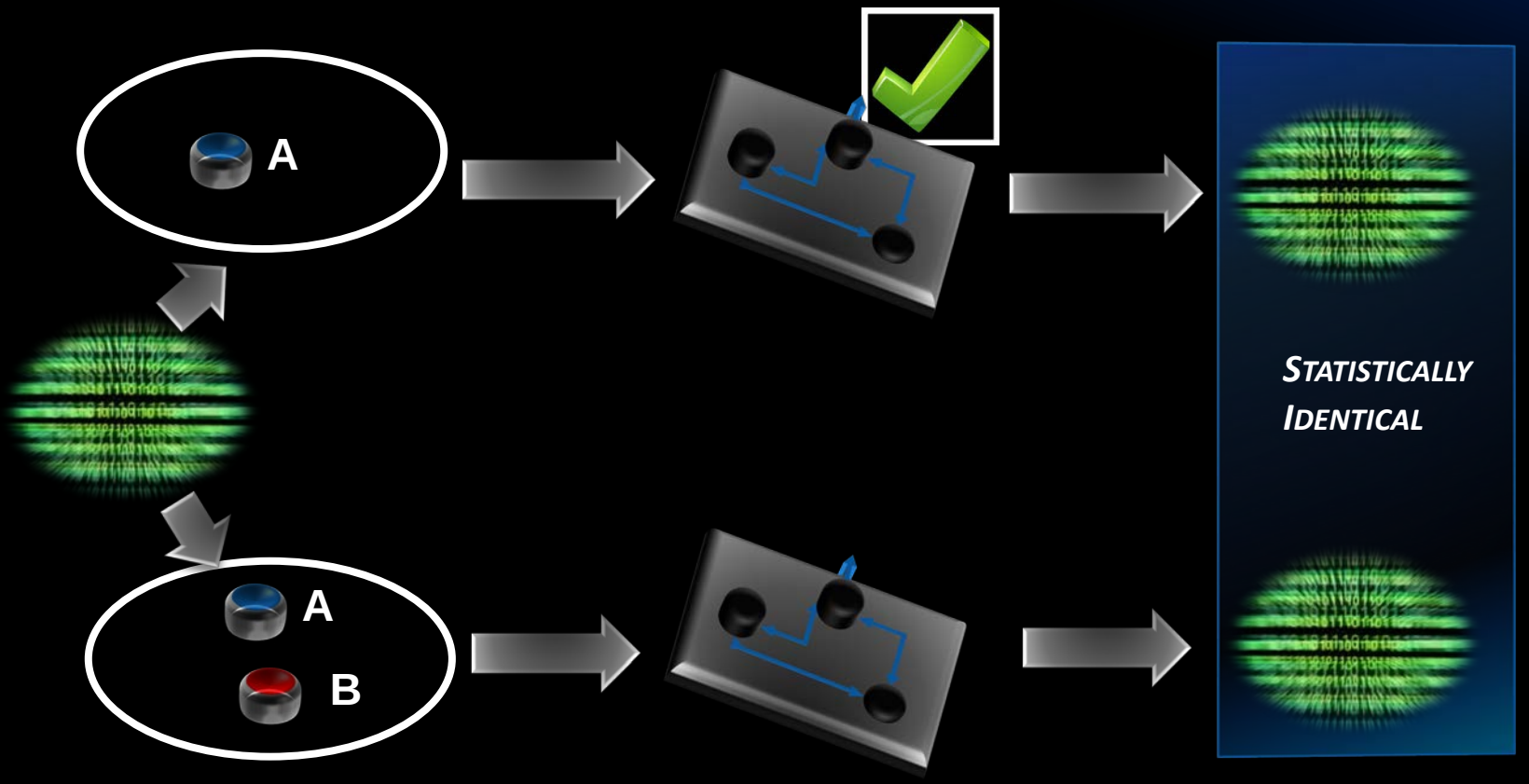
$$P(\overleftarrow{X}, \overrightarrow{X})$$

The better we can isolate the causes of natural things, the greater our understanding.



$$P(\overleftarrow{X}, \overrightarrow{X})$$


MODEL 1
A
A
B
STATISTICALLY
IDENTICAL
MODEL 2

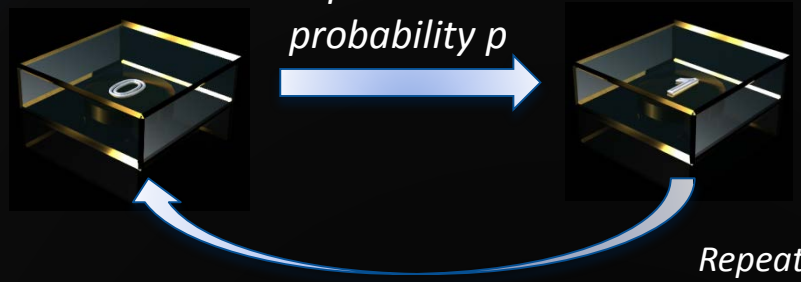


If two models have statistically identical behavior, the one that needs to keeps track of the least information (As measured by Shannon Entropy) is preferred.

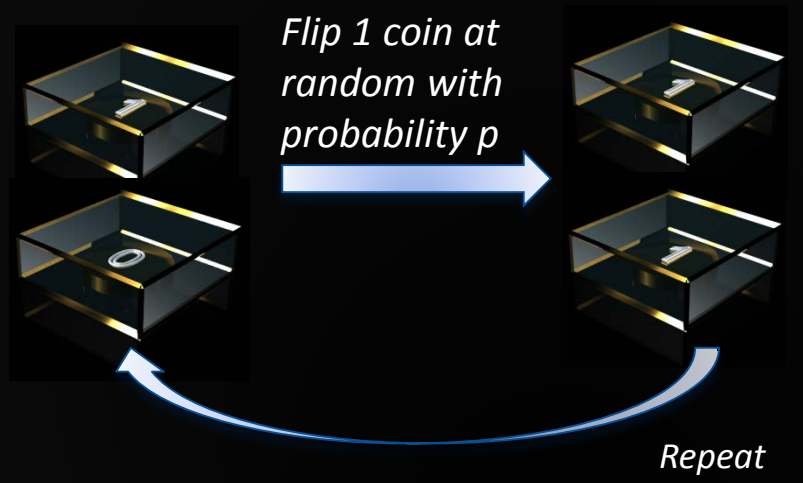
A Simple Example

Two potential models

Option A



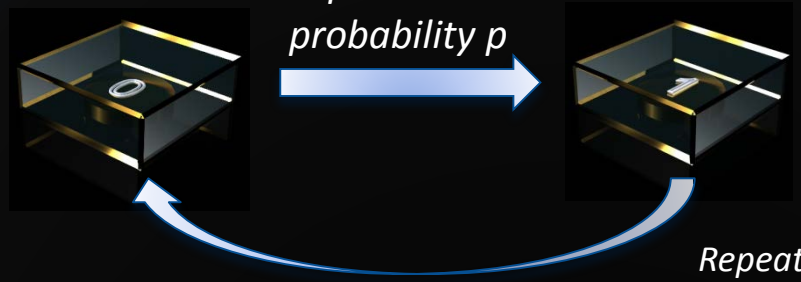
Option B



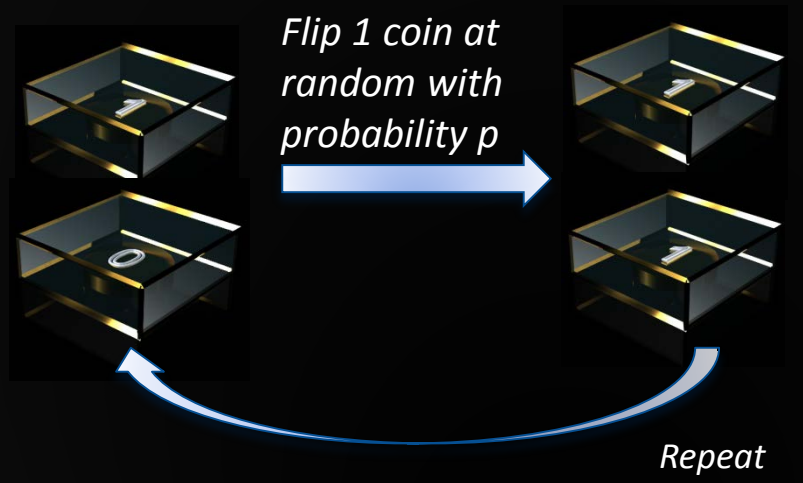
A Simple Example

Two potential models

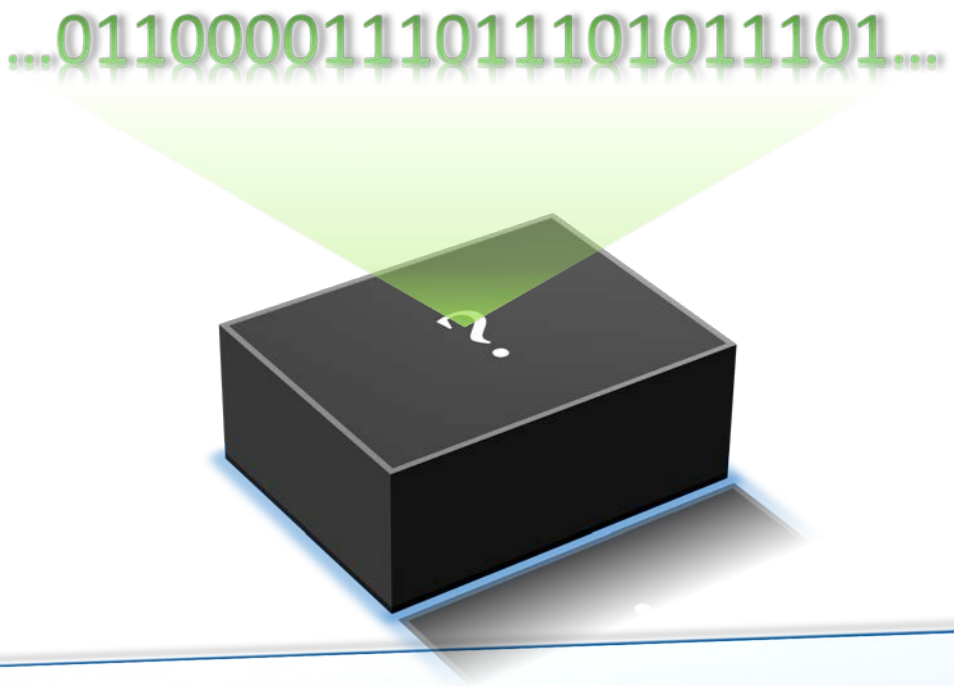
Option A



Option B

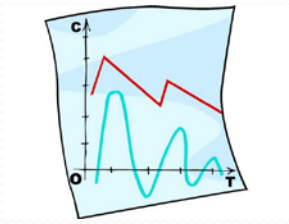


Relation to Complexity…

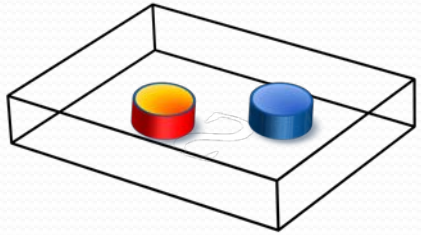


A way to quantify complexity is by the minimum amount of causes (as measured by information entropy) one must invoke to model its statistics.

Relation to Physics...

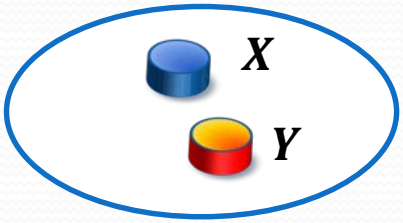


Output retrieved by appropriate measurement

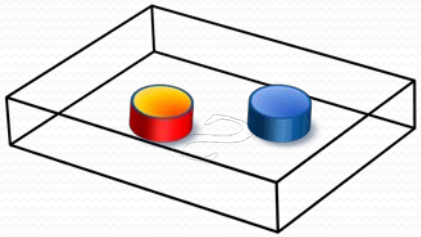


Computable Function $f(X, Y)$

Physical implementation of f



Input encoded within physical system



Mathematical Model, Input with Entropy C

Physical Simulator, Initial Entropy C

If modelling the dynamics of a phenomena requires knowledge of 'x', then any system that simulates the phenomena must store x.

That is, the system must have at least entropy

$$-\sum p_x \log p_x$$

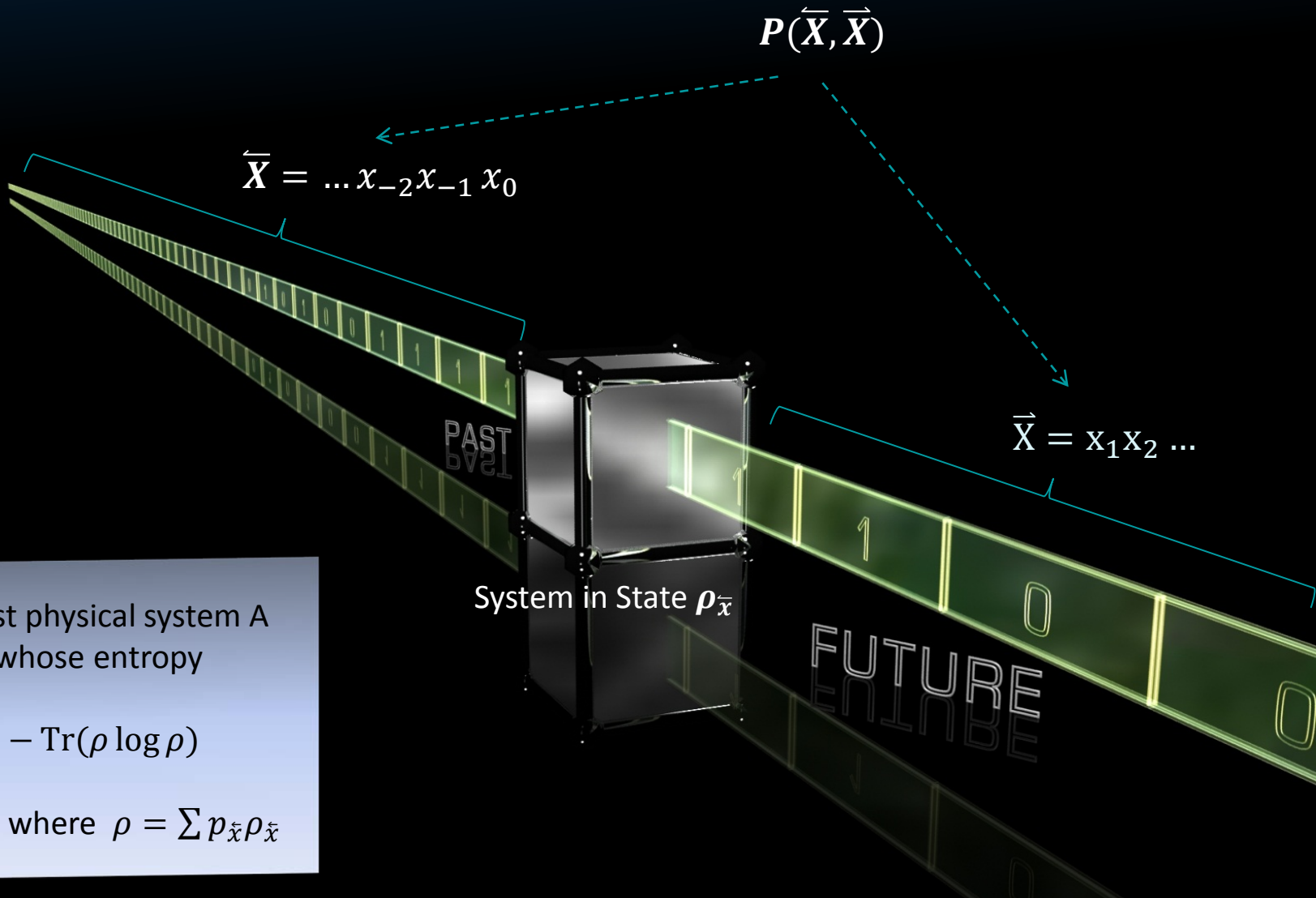
The Matrix Problem

Suppose you're a programmer for the matrix

You are tasked to program an object to simulate a particular desired behaviour.

What is the least amount of past information you must store to be able to mimic future statistics.

Stochastic Processes



Task:

Find a simplest physical system A – the system whose entropy

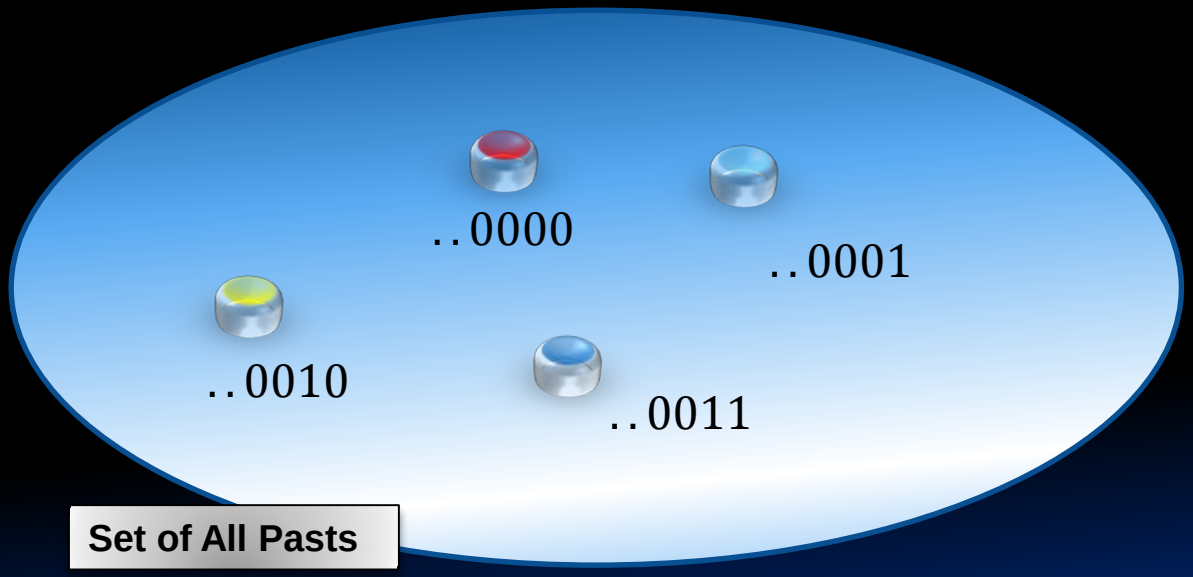
$$S(\boldsymbol{A}) = -\operatorname{Tr}(\rho \log \rho)$$

Is minimised, where $\rho = \sum p_{\overleftarrow{x}} \rho_{\overleftarrow{x}}$

Brute Force Approach..

$$\rho_{\overleftarrow{x}} = \overleftarrow{x}$$

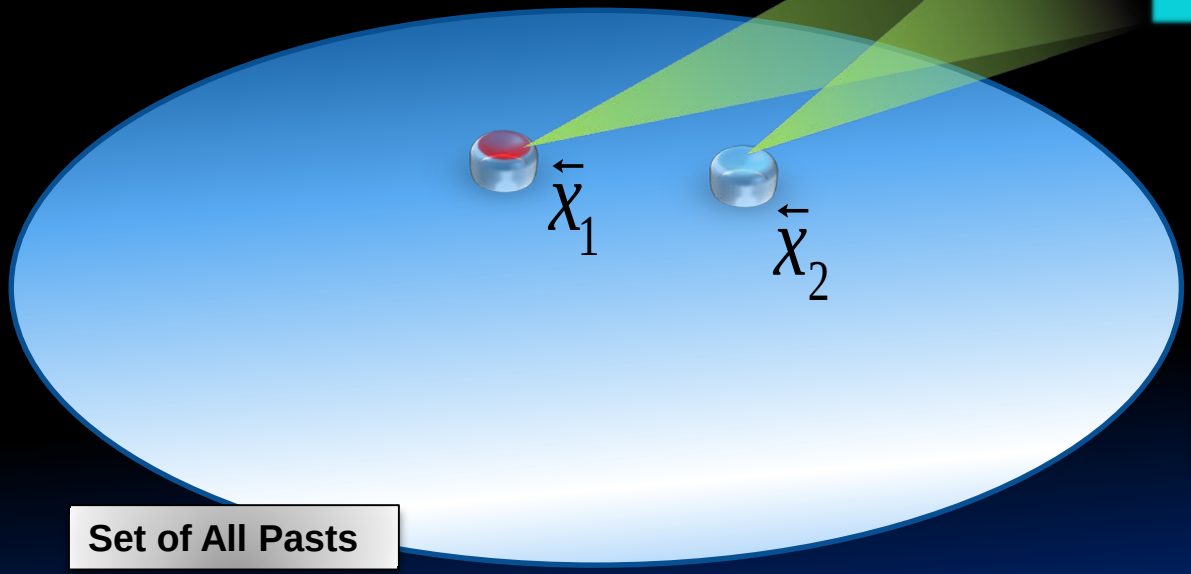
Construct a system that stores each possible past in a separate configuration.



A More Refined Method

Suppose two pasts have statistically identical futures

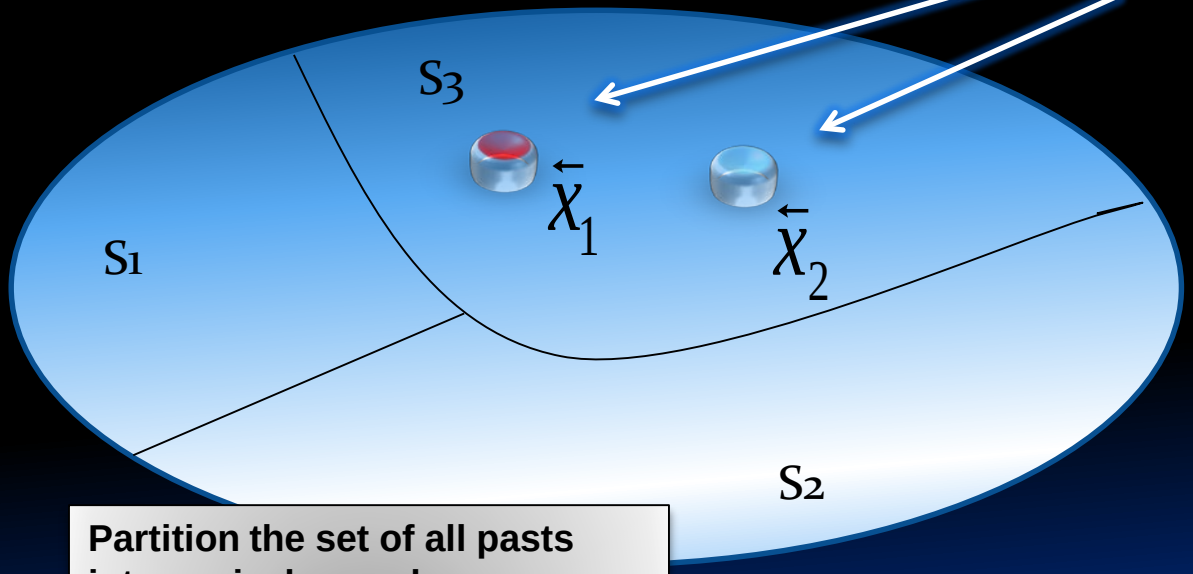
$$P(\vec{X} \mid \overleftarrow{X} = \overleftarrow{x}_1) = P(\vec{X} \mid \overleftarrow{X} = \overleftarrow{x}_2)$$



The information needed to distinguish the two is irrelevant to the future of the process and can thus be discarded.

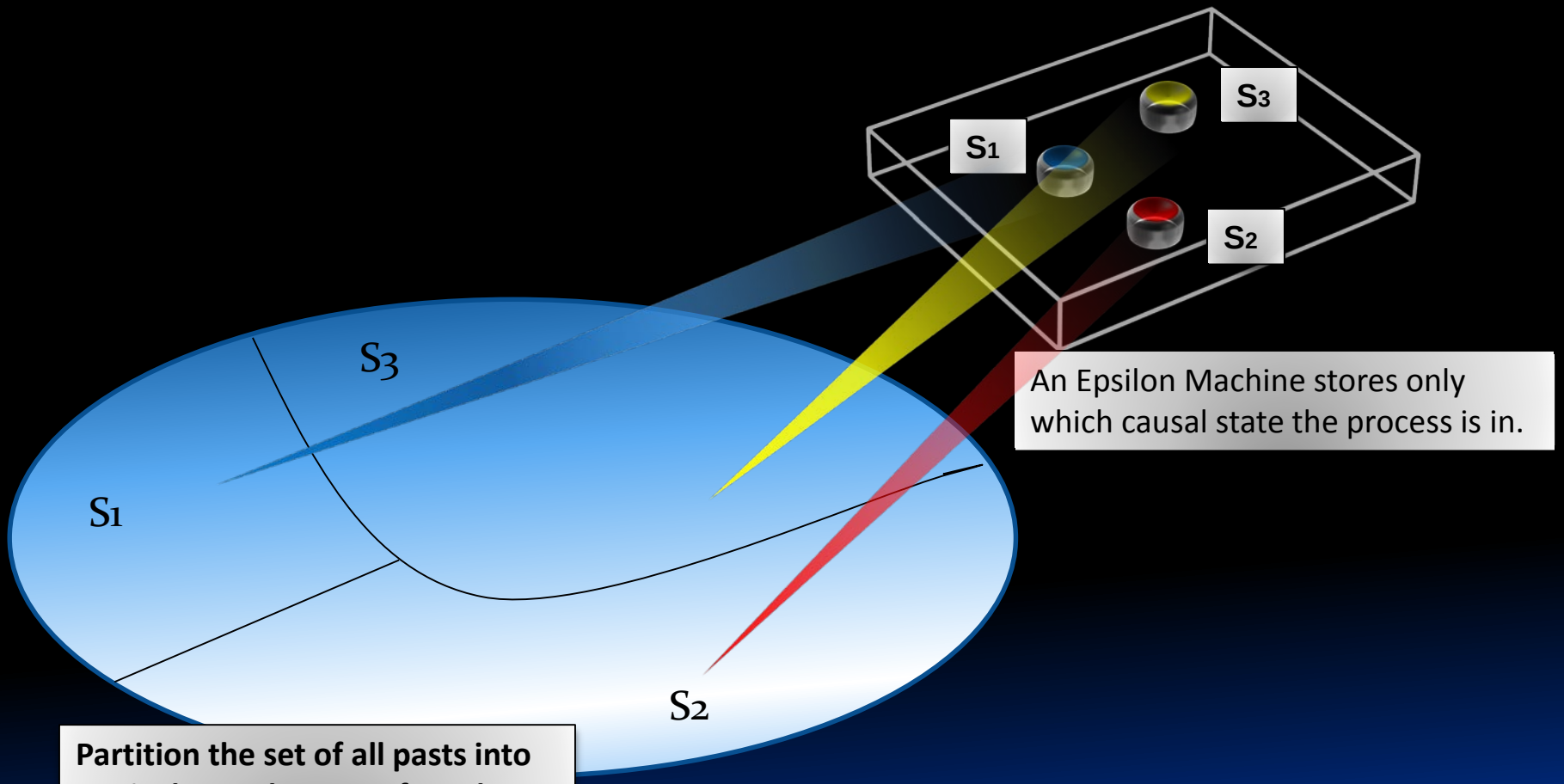
A More Refined Method

Two different possible pasts belong to the same Causal state if they have coinciding futures.



Partition the set of all pasts into equivalence classes, referred to as Causal states.

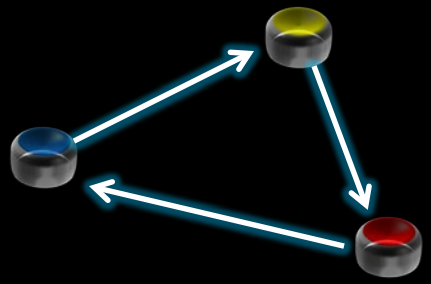
A More Refined Method

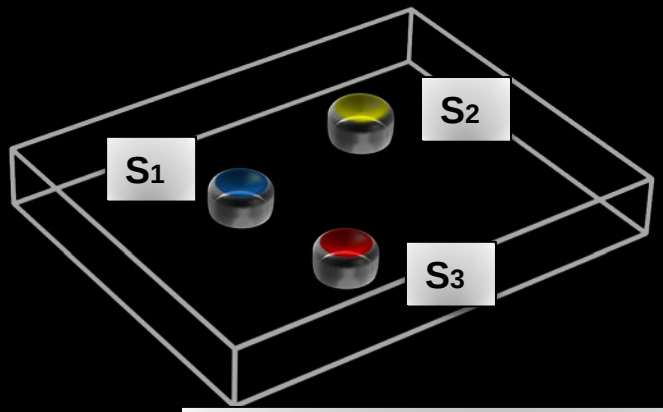


An Epsilon Machine stores only which causal state the process is in.

Partition the set of all pasts into equivalence classes, referred to as Causal states.

The Simplest Model… Epsilon Machines





An Epsilon Machine stores only which causal state the process is in.

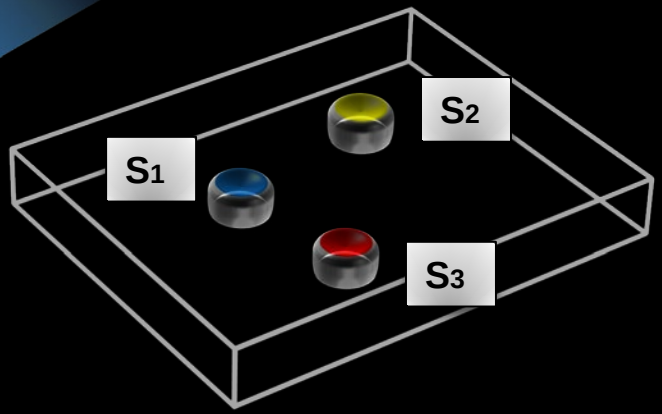
The Stochastic Process can then be completely defined by transition probabilities on the causal states.

$$T^{r}_{j,k}$$

Probability a Stochastic process in Causal state S_j will emit output `r' and transition to S_k

The Simplest Model...

Probability the process is in Causal State S_i



Internal Entropy

$$C_\mu = -\sum p_i \log p_i$$

To simulate a sequence of random coin flips....

We have a process with exactly 1 Causal State

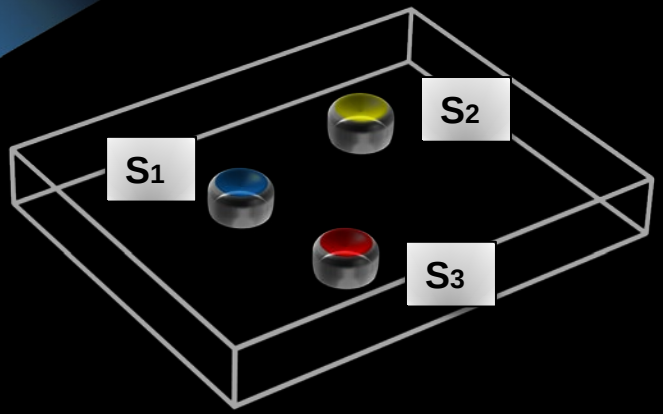
No Information about the Past is required!

Statistical Complexity

Probability the process is in Causal State S_i

Internal Entropy

$$C_\mu = -\sum p_i \log p_i$$

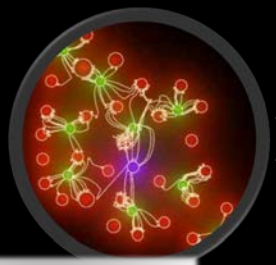


No classical system can simulate a given stochastic process using less information than a Epsilon Machine

Crutchfield, Young, Phys. Rev. Lett. 63, 105–108 (1989)

C_μ Is a **intrinsic** property of a stochastic process that measures the minimal amount of memory required to simulate the given process.

Statistical Complexity



IEEE Trans. Neural Networks, 10, 2, 284-302

Neural Networks

Applied to wide range of systems.

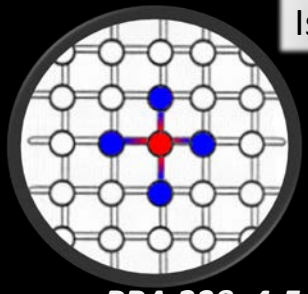
Pseudo-random Number generators.

Physica A: 356, 1, 133-138

Dripping Faucets

Physica A: 257, 1-4, 385-389

Ising Models



PRA 238, 4-5, 244-252

No classical system can simulate a given stochastic process using less information than a Epsilon Machine

Crutchfield, Young, Phys. Rev. Lett. 63, 105–108 (1989)

C_μ Is a **intrinsic** property of a stochastic process that measures the minimal amount of memory required to model a given process.

HIGH COMPLEXITY!

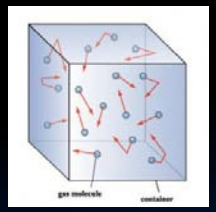
ORDERED

LOW COMPLEXITY

RANDOM

LOW COMPLEXITY

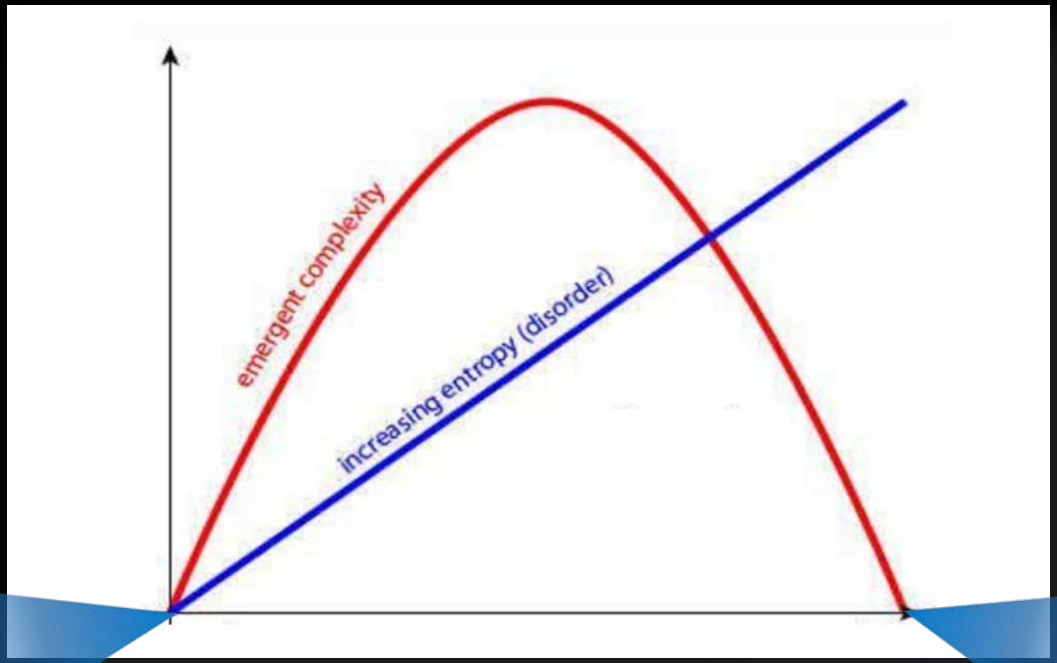
PENDULUM



IDEAL GAS

Image Courtesy of Sean Carol, 'Setting time Aright' 2011

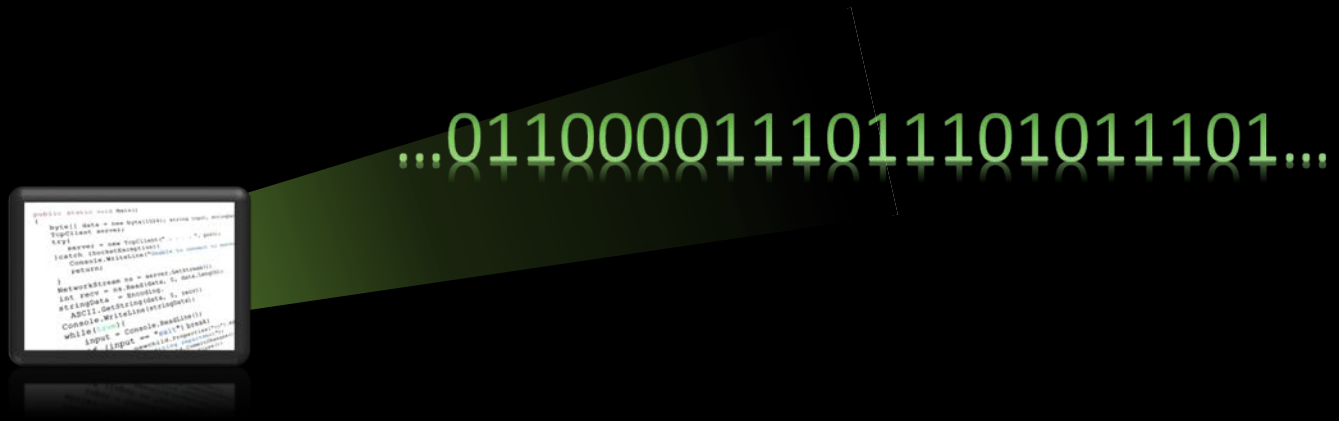
LOW COMPLEXITY HIGH COMPLEXITY LOW COMPLEXITY



LOW DISORDER

HIGH DISORDER

Kolomogorov Complexity



Kolmogorov Complexity

The ‘size’ of the minimal computer program that can be used to generate a specific observed sequence

Non-Computable

Maximum for Random Sequences.

Kolmogorov Complexity

"[Kolmogorov Complexity] plays a very important role in every discussion of measuring complexity: in a pious act of homage to our intellectual ancestors, it is solemnly taken out, exhibited, and solemnly put away as useless for any practical application."

- Cosma Shalizi

Algorithmic Information

The 'size' of the minimal computer program that can be used to generate a specific observed sequence

Non-Computable

Maximum for Random Sequences.

Direct Approaches...

$$C \propto Entropy * (1 - Entropy)$$

Computable

Exhibits Desired Behaviour

Don't Directly Say Anything about Structure

Lopez-Ruiz, Ricardo, Hector Mancini, and Xavier Calbet. "A statistical measure of complexity." *Phys Lett A* 209 (1995): 321-326.

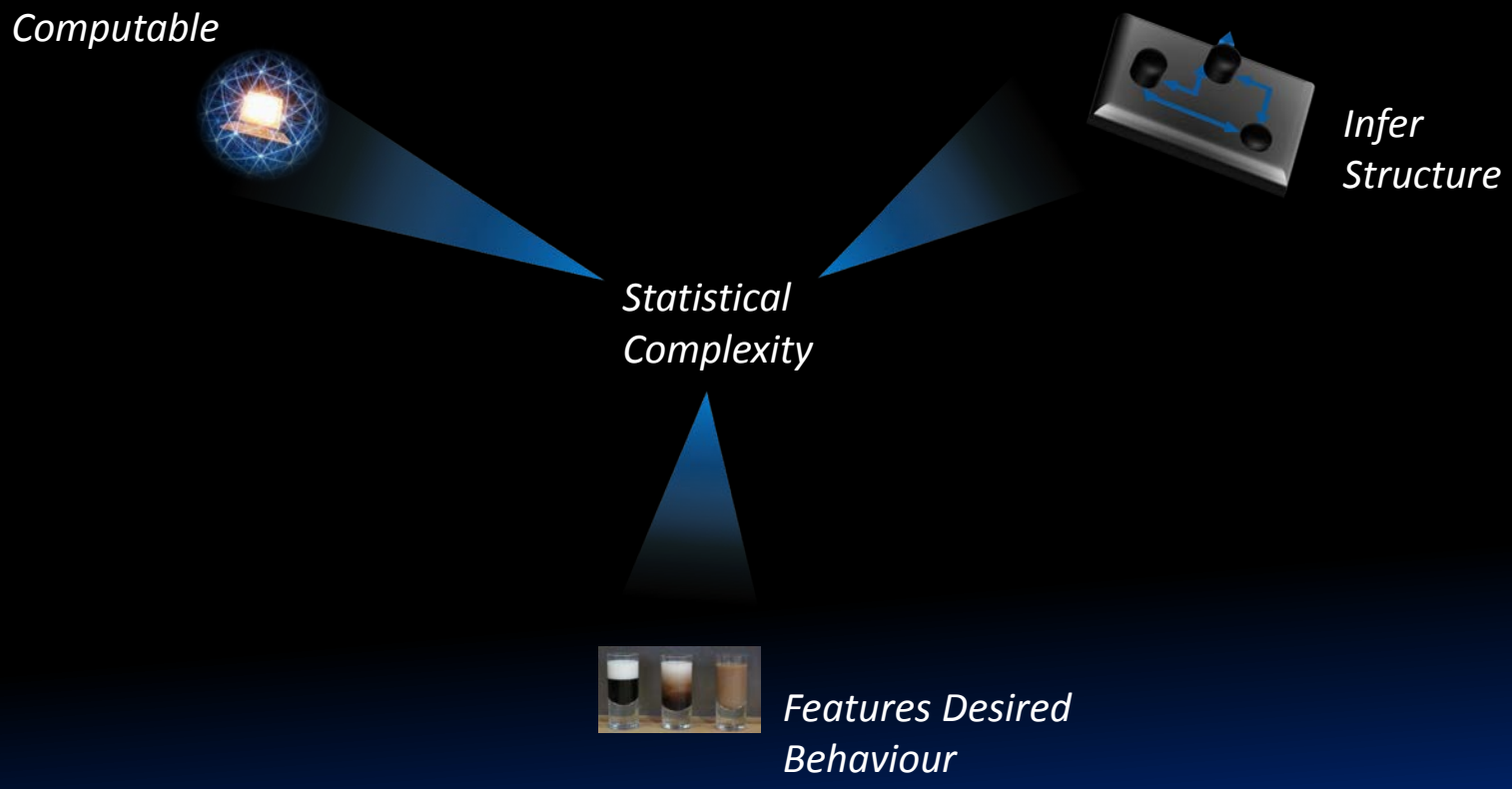
Computable

Desired Behaviour

Infers Structure

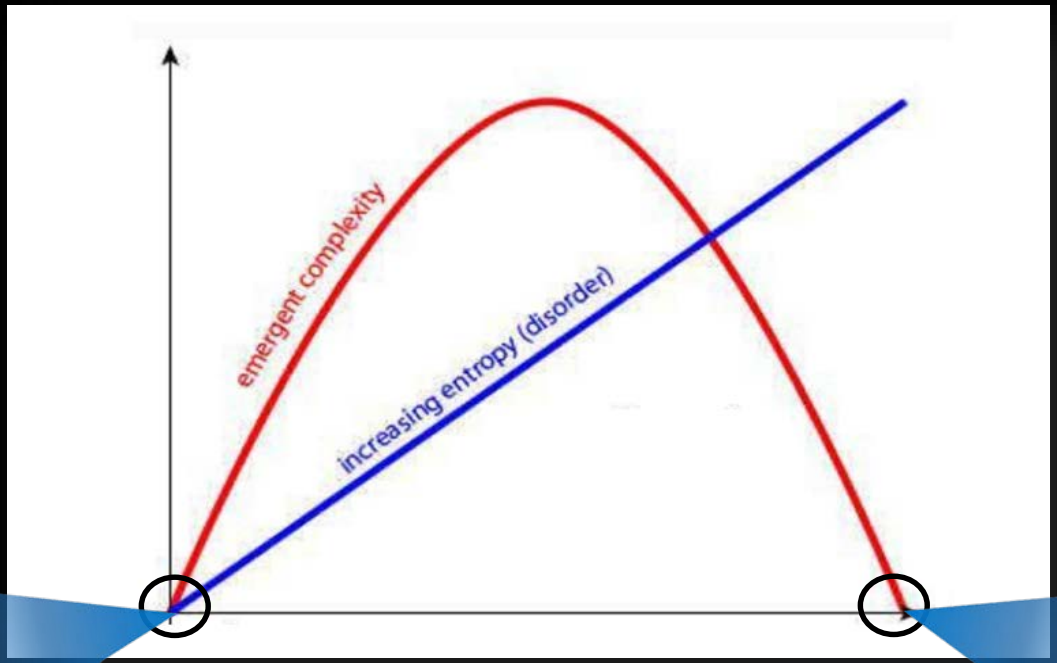
	Computable	Desired Behaviour	Infers Structure
Algorithmic Information	✗	✗	✓
LMC Complexity	✓	✓	✗
Logic Depth	✗	✓	✓
Effective Measure Complexity	✗	✓	✓
Thermodynamic Depth	?	✗	✓

Statistical Complexity



Statistical Complexity

Low Complexity | High complexity | Low Complexity

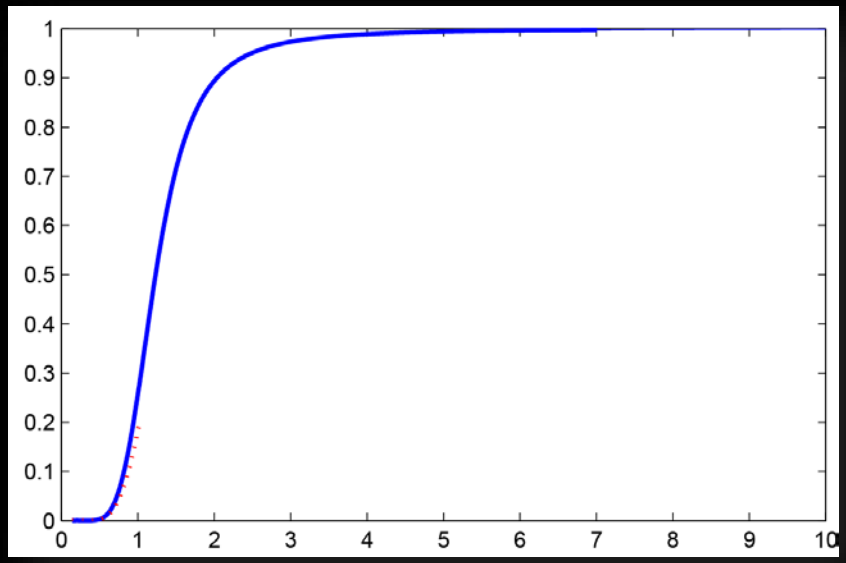


Statistical Complexity = 0 For uniform process

Statistical Complexity = 0 for random sequence.

State of Infinite Spin Chain at Temperature T

STATISTICAL COMPLEXITY

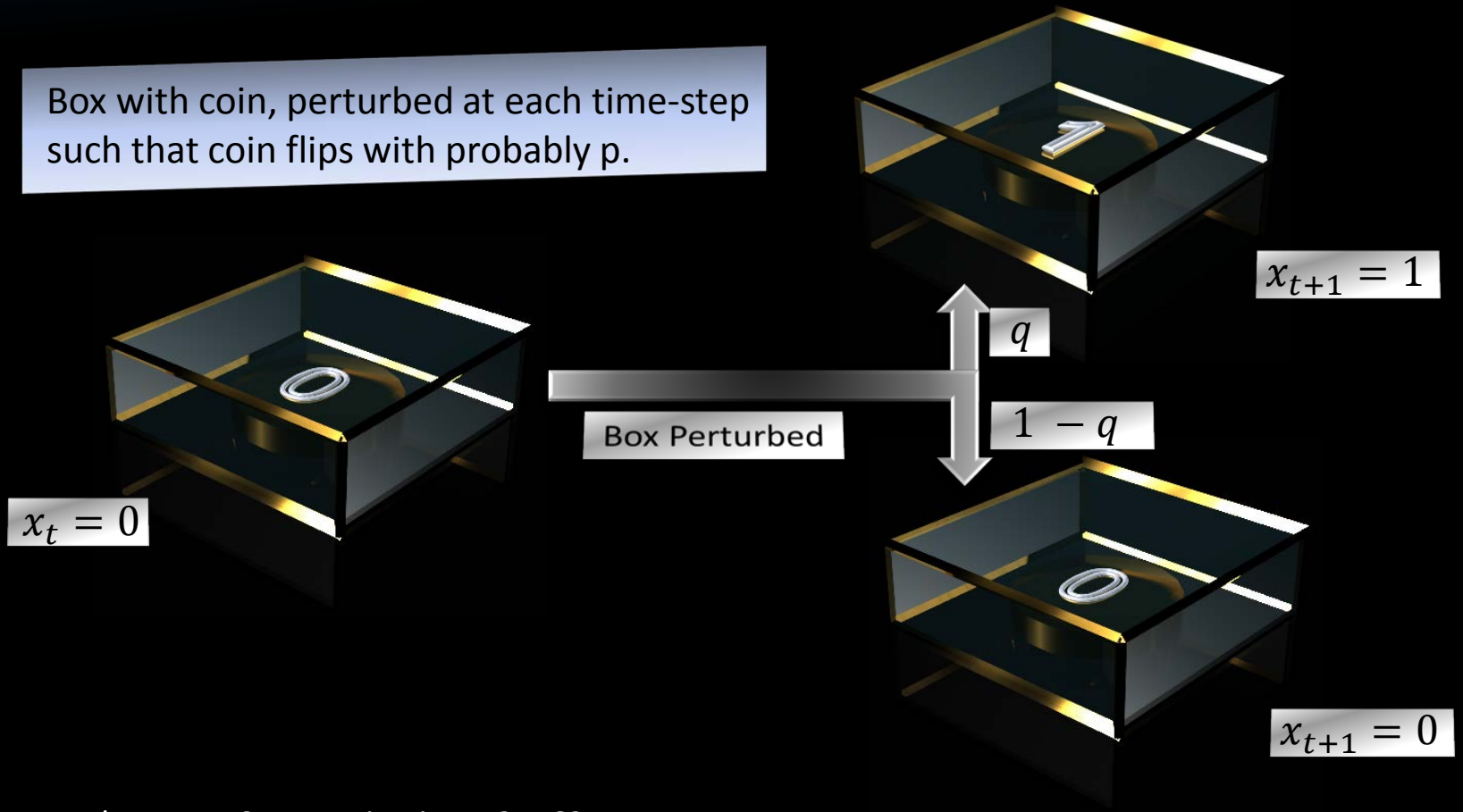


TEMPERATURE

Crutchfield, J., Feldman, D., "Statistical complexity of simple one-dimensional spin systems." Physical Review E 55.2 (1997): R1239.

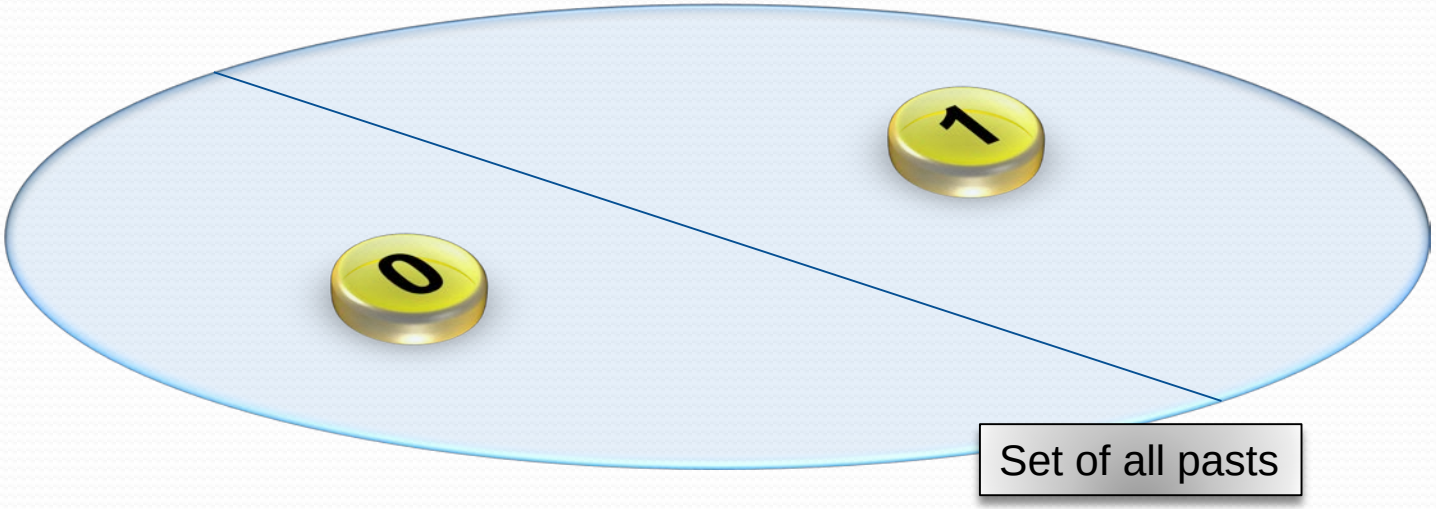
Case Study: The Perturbed Coin

Box with coin, perturbed at each time-step such that coin flips with probably p.



Gu. et. al. Nature Communications, 3, 762

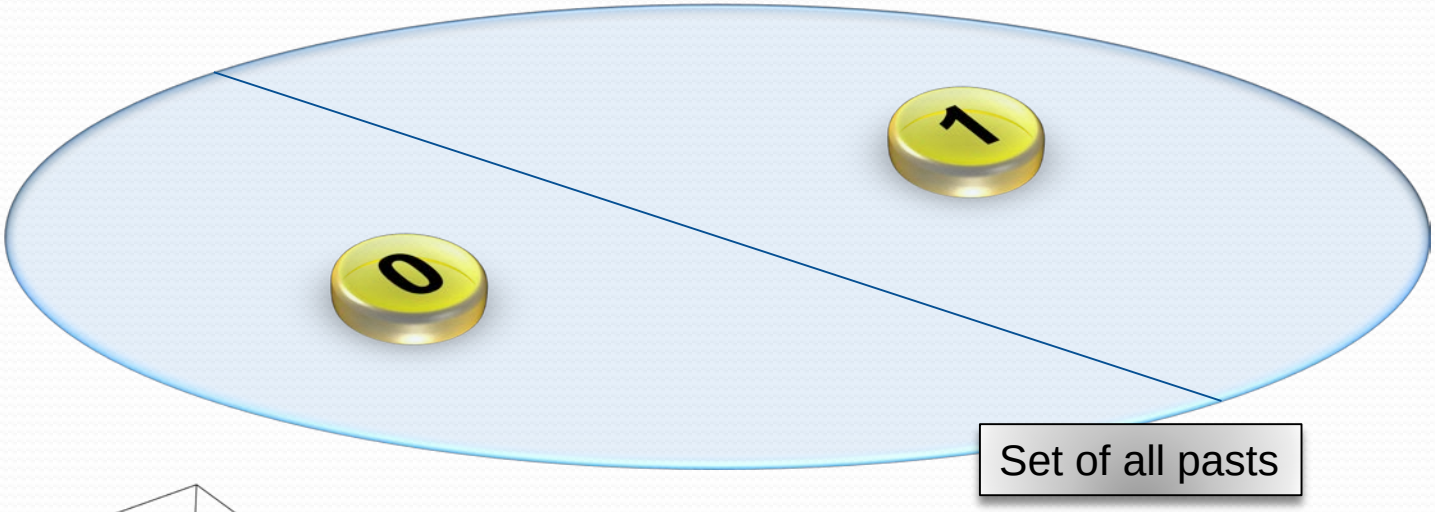
Case Study: The Perturbed Coin

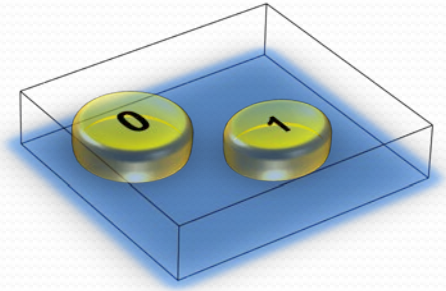


$$P(\vec{X}|0) \neq P(\vec{X}|1)$$

We cannot discard information about the state of the coin.

Case Study: The Perturbed Coin

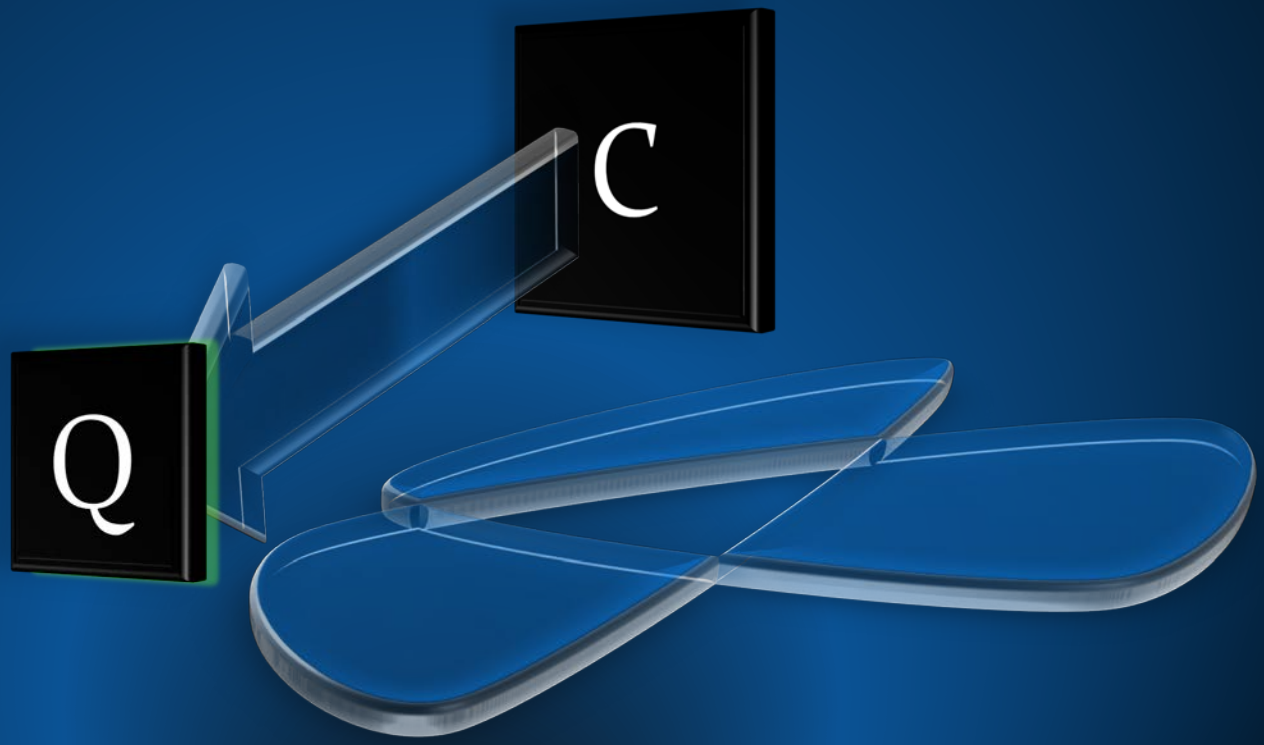




Optimal classical system for generating $\mathbf{P}(\overleftarrow{\mathbf{X}}, \overrightarrow{\mathbf{X}})$
has internal entropy 1, for any q ≠ 0.5

But as q→ 0.5, the process tends towards a completely random sequence!

Can Quantum Logic Provide Simpler Understanding?

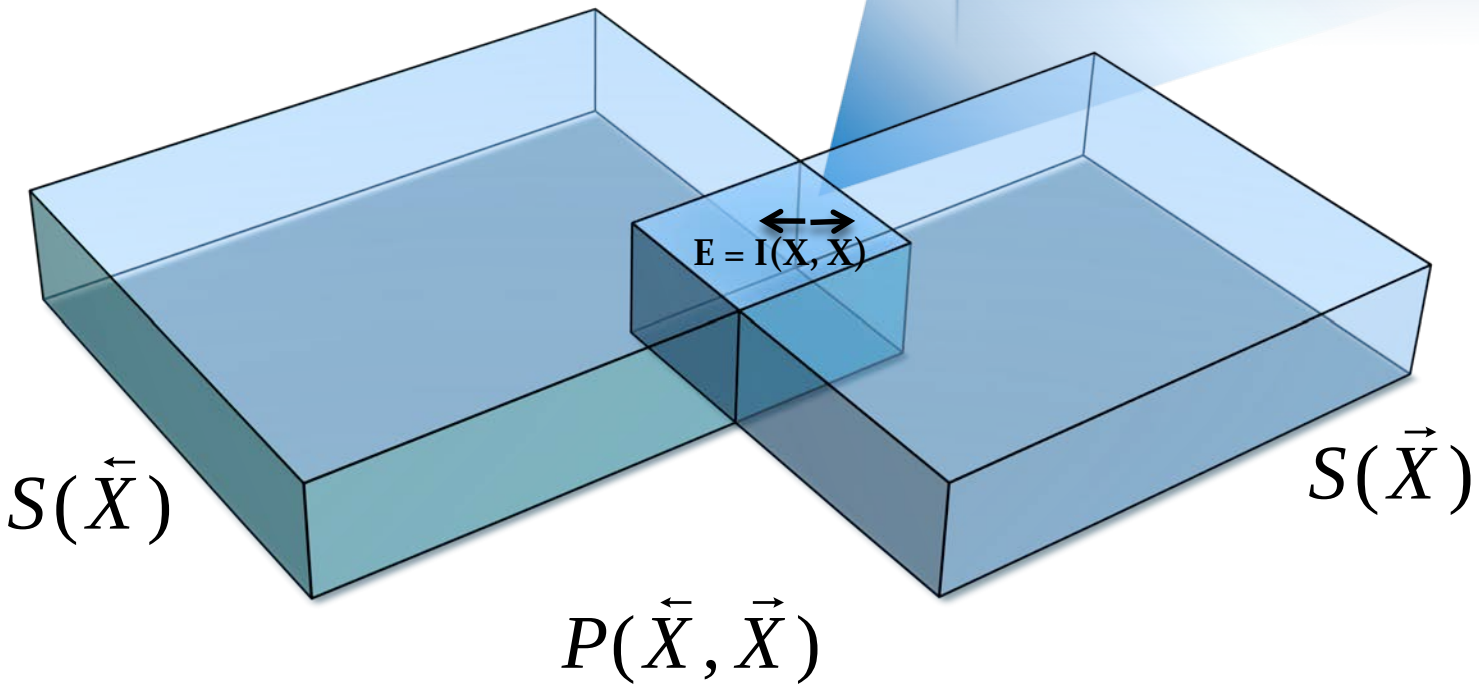


Can Epsilon Machines be improved?

Knowledge of the past contains

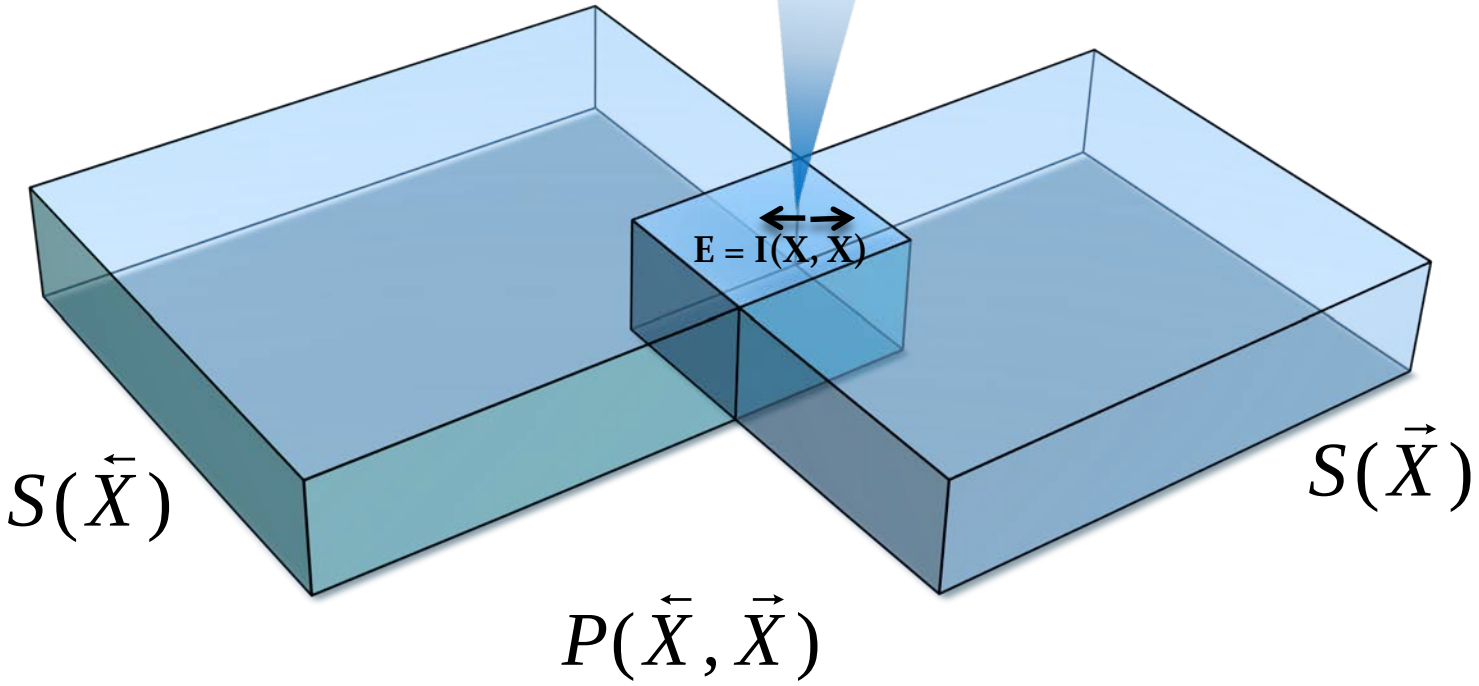
$$S(\vec{X}) - S(\vec{X} \mid \overleftarrow{X}) = I(\vec{X} \mid \overleftarrow{X})$$

bits about the future.



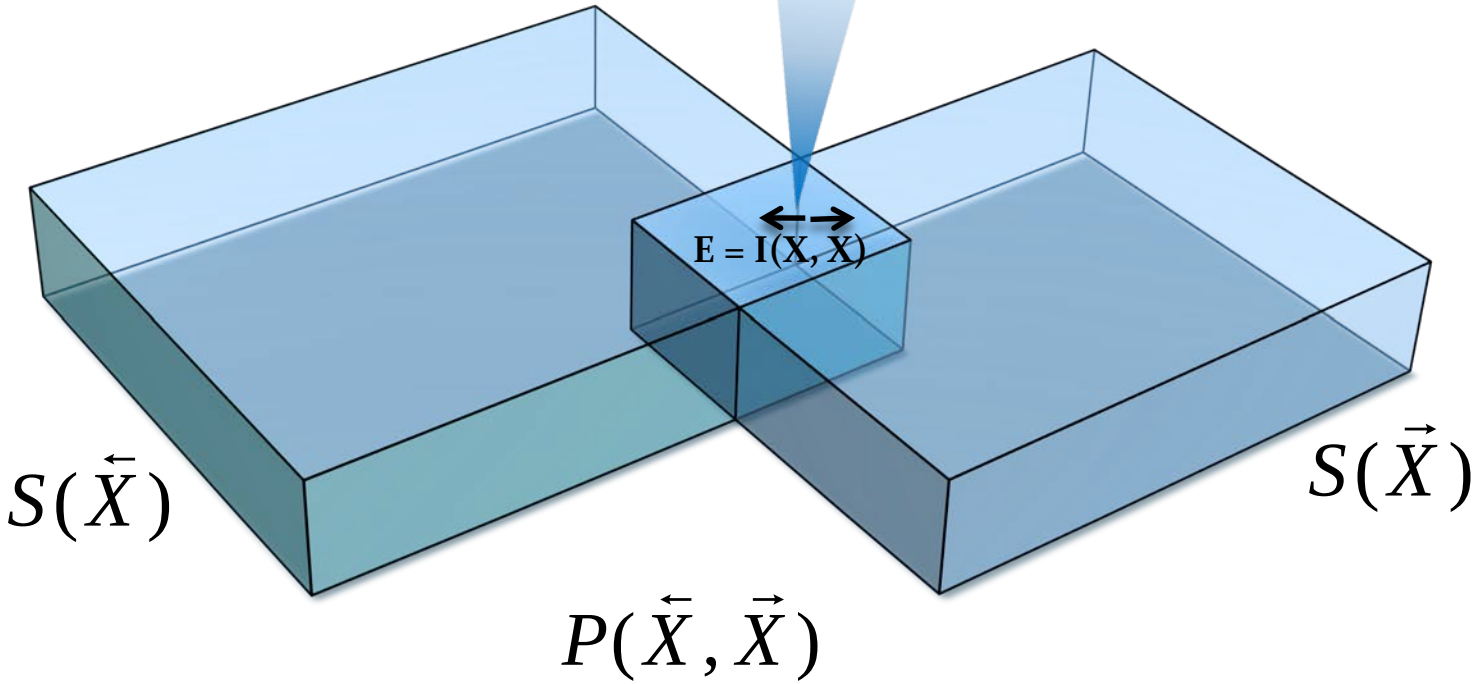
Can Epsilon Machines be improved?

Lower bound on how many bits any simulator of The process must store.



Can Epsilon Machines be improved?

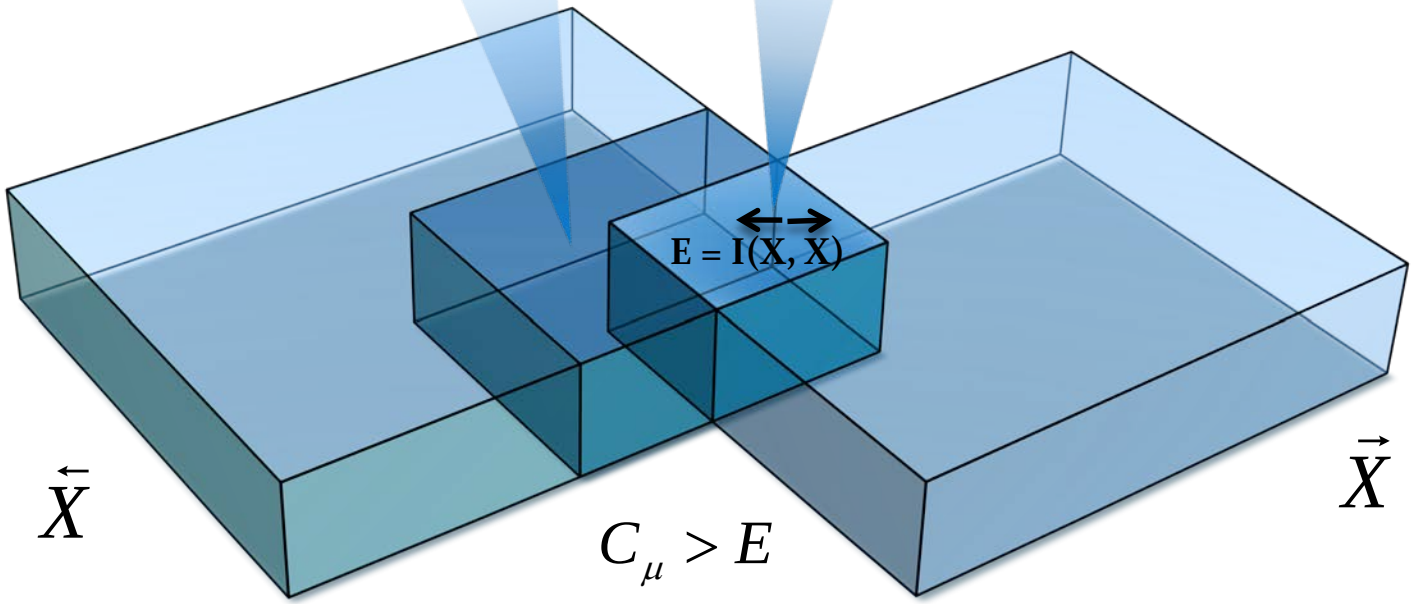
Why should a simulator store any more information than this?



Can Epsilon Machines be improved?

An epsilon machines must store C_μ bits to know to retain all information About the future..

Lower bound on how many bits any simulator of The process must store.

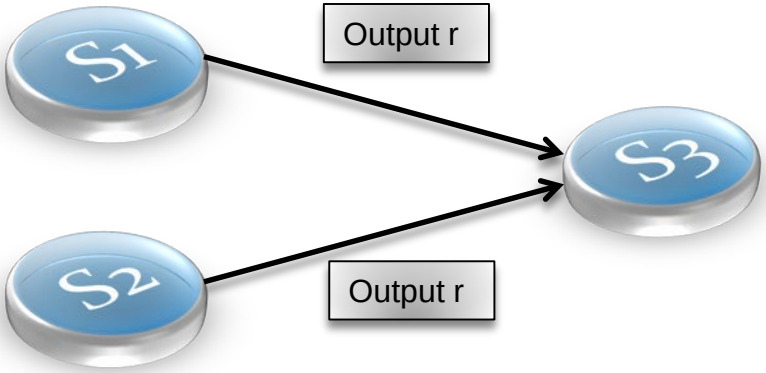


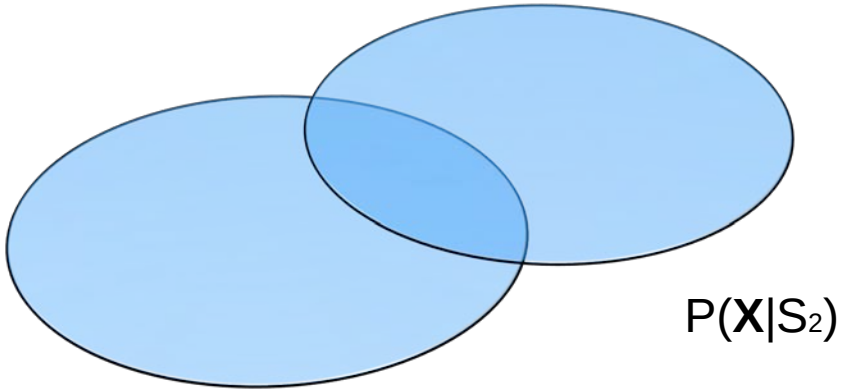
JP **Crutchfield**, CJ Ellison, "Time's Barbed Arrow." Phys. Rev. Lett 2009

Some of the information recorded is still not relevant to the future – the Epsilon machine still wastes information!

The Source of Extraneous Cause

Suppose two differing causal states have finite probability to transition to an coinciding causal state after coinciding output.





The future of the two causal states are not entirely distinct.

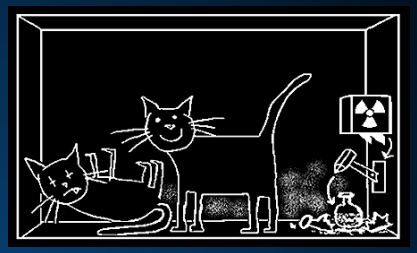
A classical epsilon must store a property A that distinguishes S_1 and S_2. But observation of the entire future does not guarantee the ability to retrieve A.

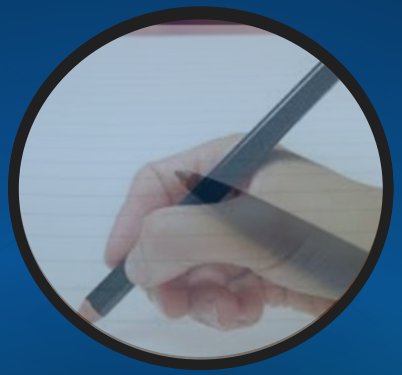
Some of the storage used to keep track of A is wasted.

Entropy = 1

Entropy = 0

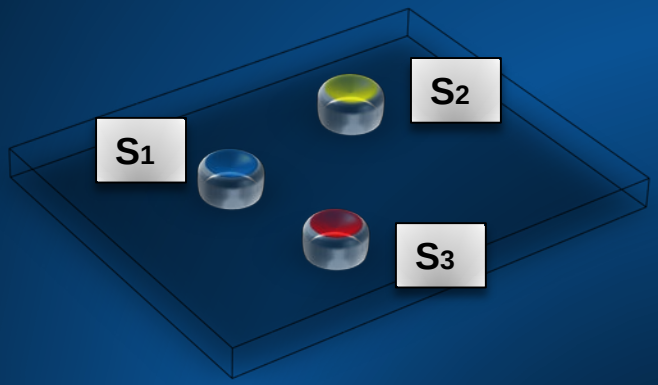
A quantum model can perform superposition of both!





Entropy = 1

Entropy = 0



Classical models must allocate enough storage to distinguish every causal state

Quantum systems can go beyond this by compressing the information further… distinguishing the causal states only to the degree that they affect the future.

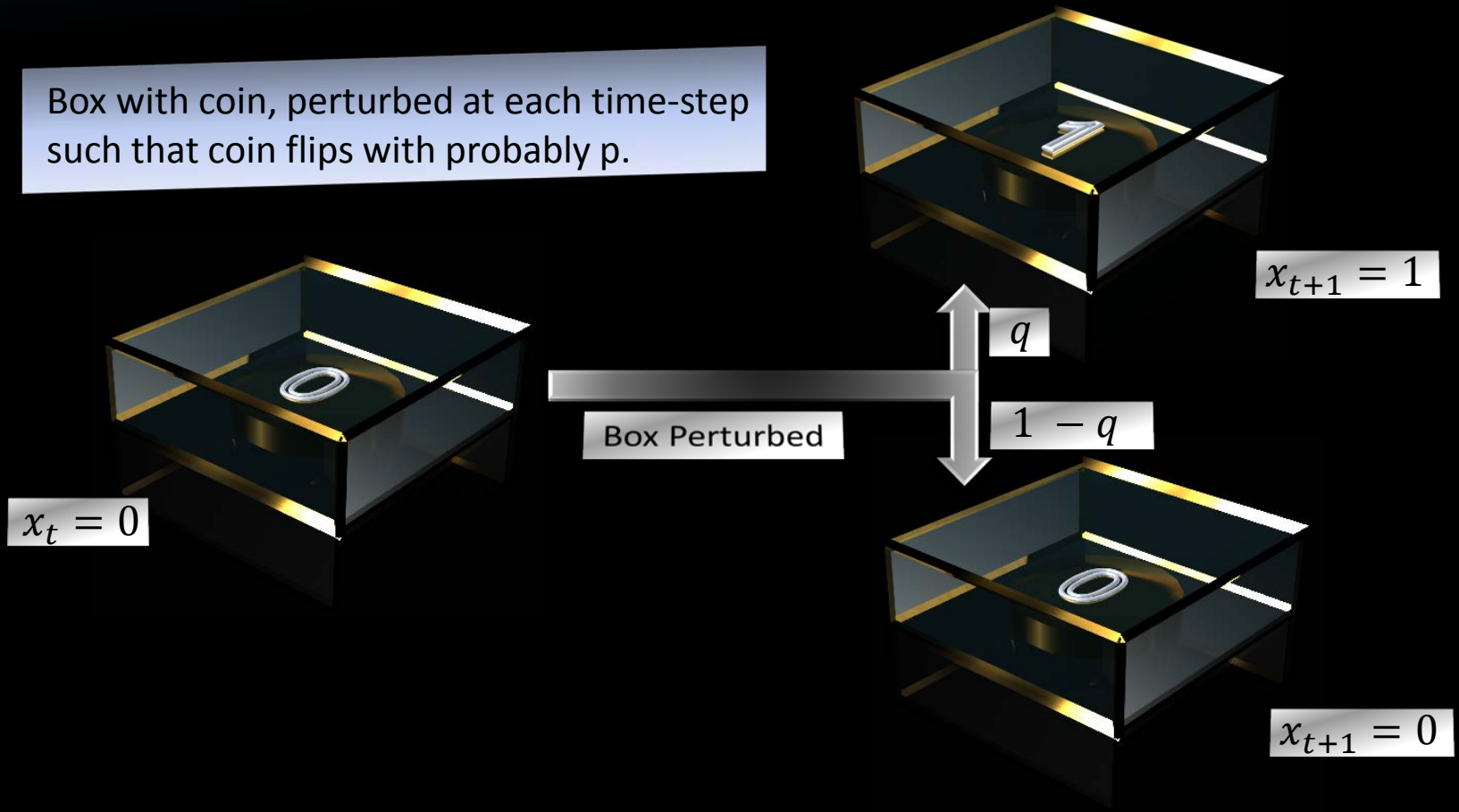
THEOREM

Provided the best classical model for a stochastic process stores some unnecessary information, there exists a simpler quantum model

M.Gu, K.Wiesner, E.Rieper, V.Vedral, Nature Communications, 3, 762

Back to the Perturbed Coin....

Box with coin, perturbed at each time-step such that coin flips with probably p.



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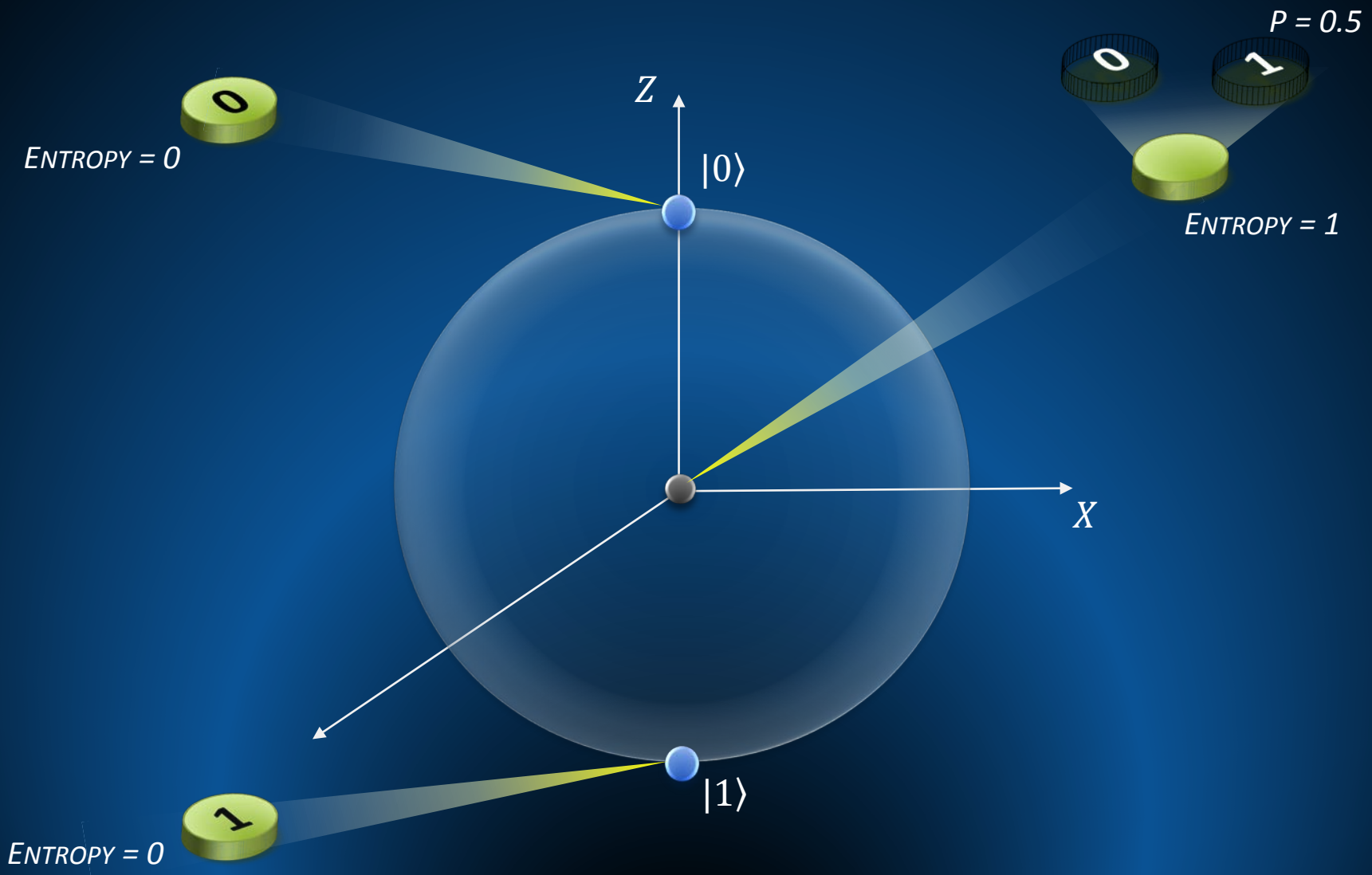
Classical models always require us to perfectly distinguish *Between 0 and 1!*

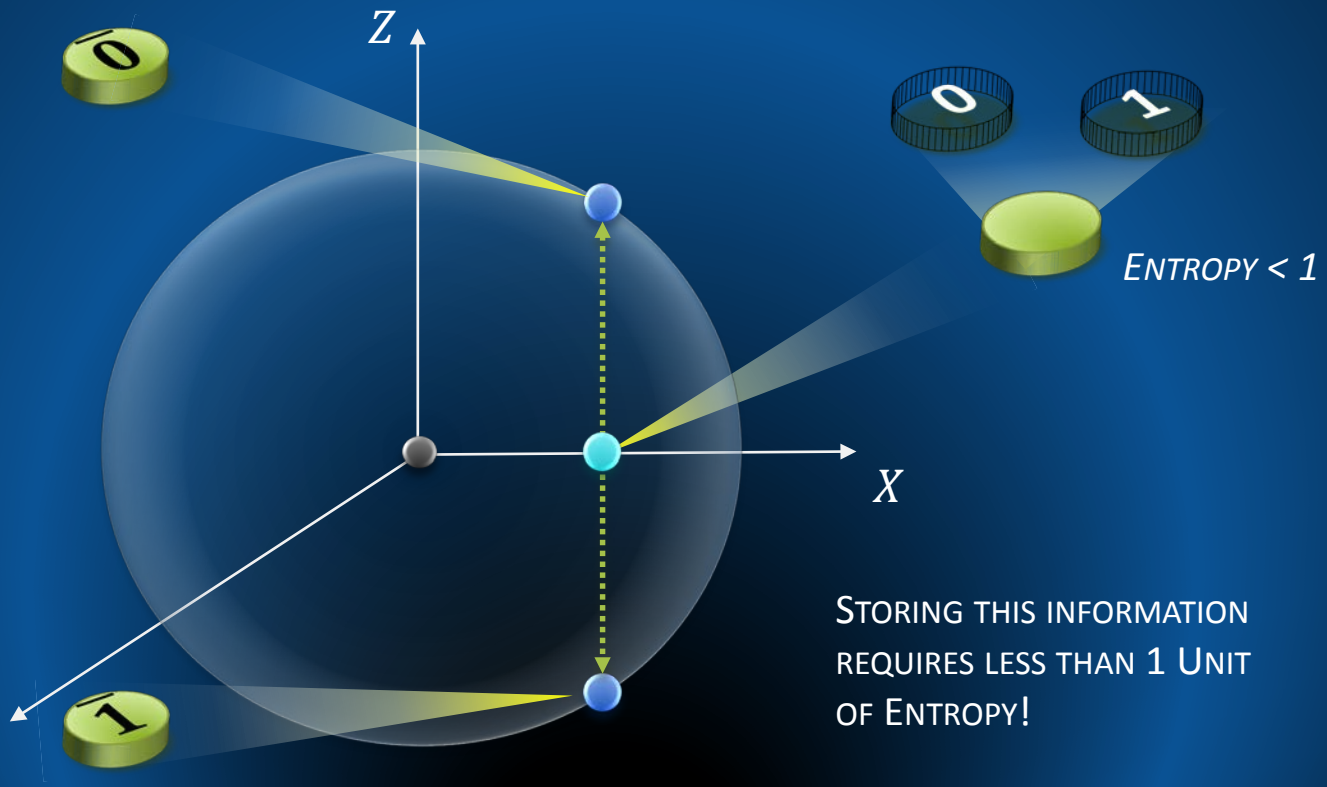


Classical models always require us to perfectly distinguish *Between 0 and 1!*

Simplifying with Quantum

Encode as $|\bar{0}\rangle = \sqrt{q}|0\rangle + \sqrt{1-q}|1\rangle$

Encode as $|\bar{1}\rangle = \sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$



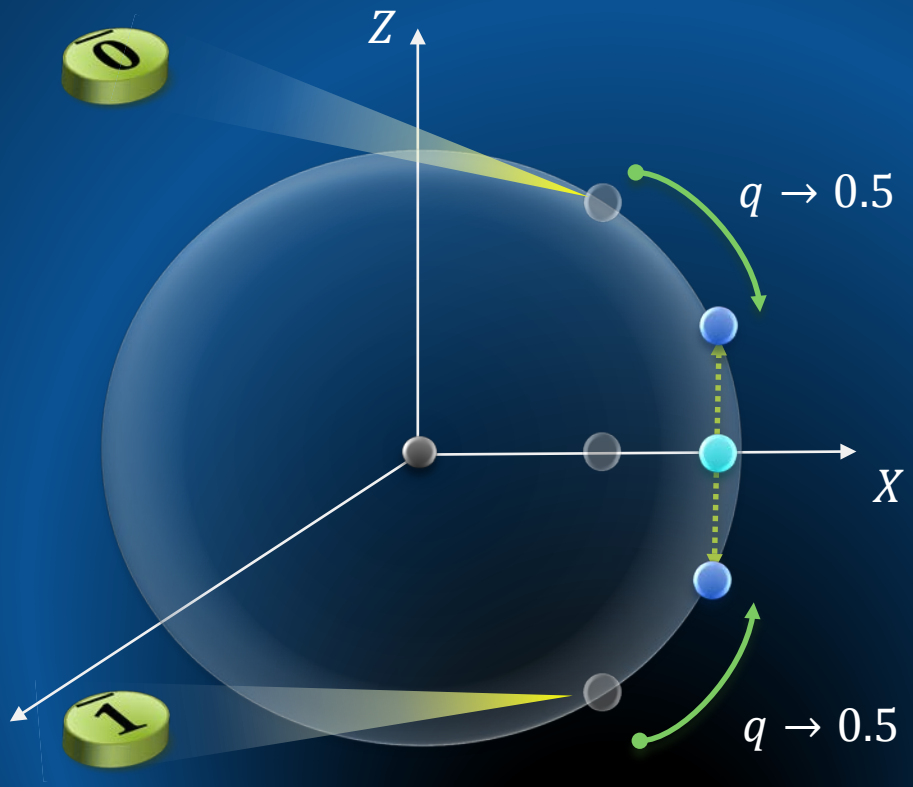
Entropy < 1

Storing this information requires less than 1 Unit of Entropy!

Simplifying with Quantum

Encode as $|\bar{0}\rangle = \sqrt{q}|0\rangle + \sqrt{1-q}|1\rangle$

Encode as $|\bar{1}\rangle = \sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$

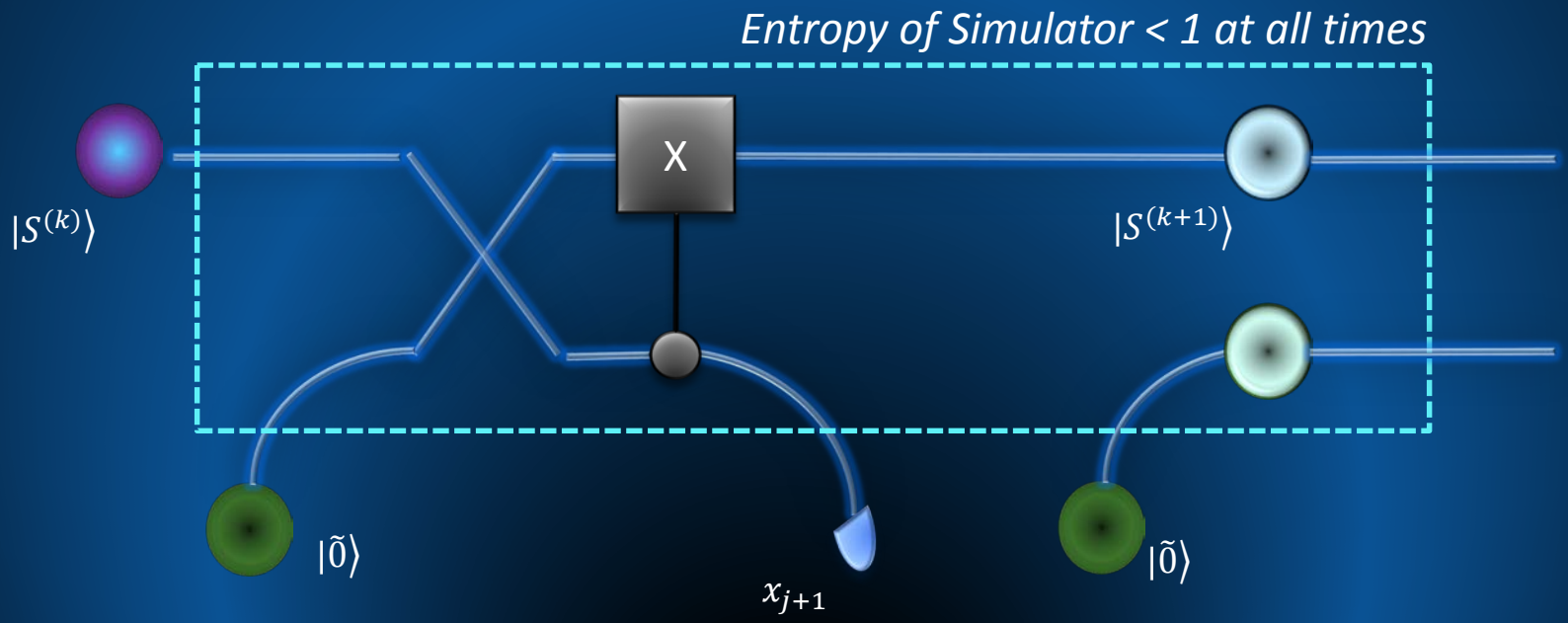


As $q \to 0.5$, the quantum model takes asymptotically no memory.

Simplifying with Quantum

Encode 0 as $|\bar{0}\rangle = \sqrt{q}|0\rangle + \sqrt{1-q}|1\rangle$

Encode 1 as $|\bar{0}\rangle = \sqrt{1-q}|0\rangle + \sqrt{q}|1\rangle$



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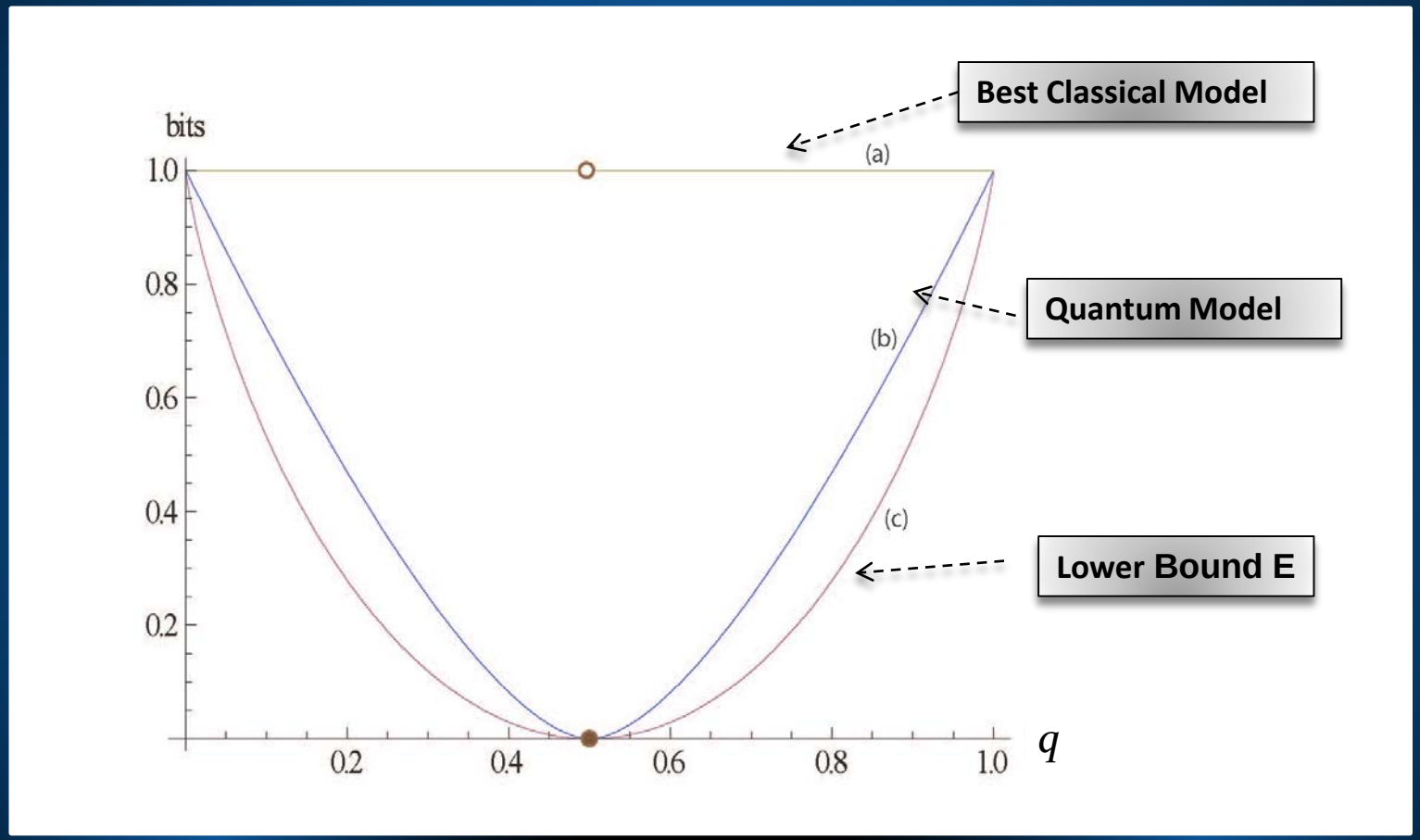
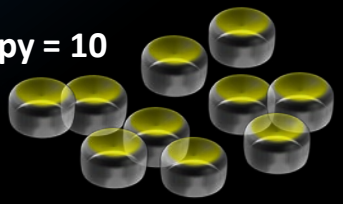


Perturbed Coin Lattice

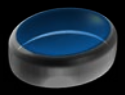
Example: K = 10, q = 0.4

Entropy = 10

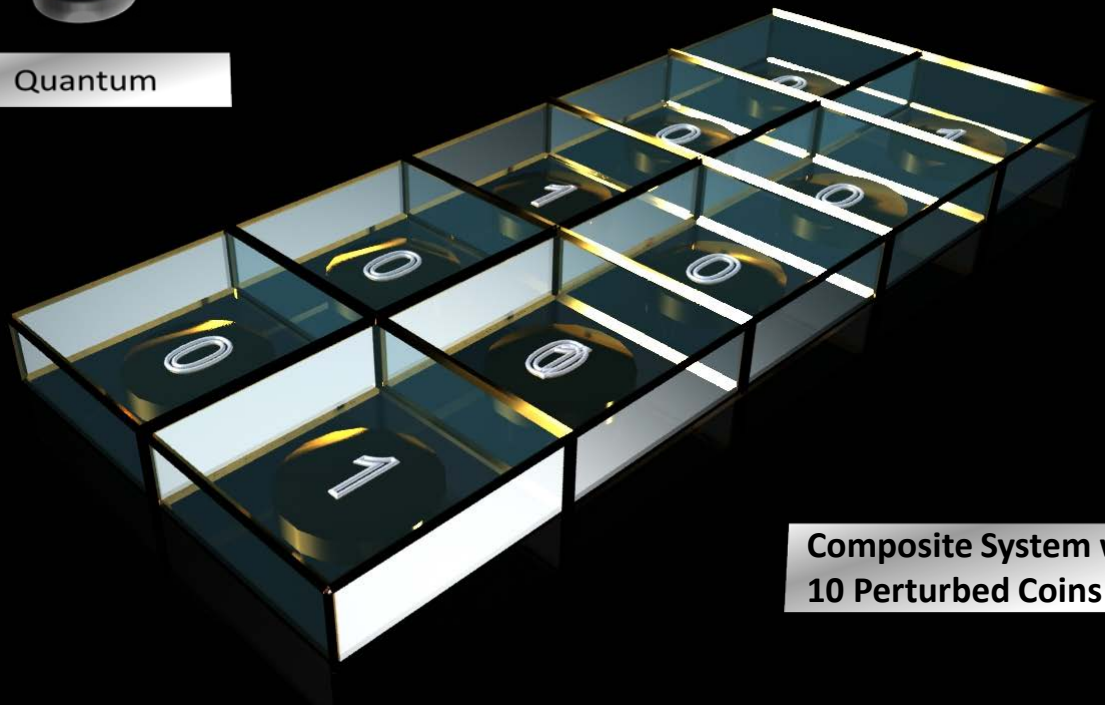


Classical

Entropy < 1

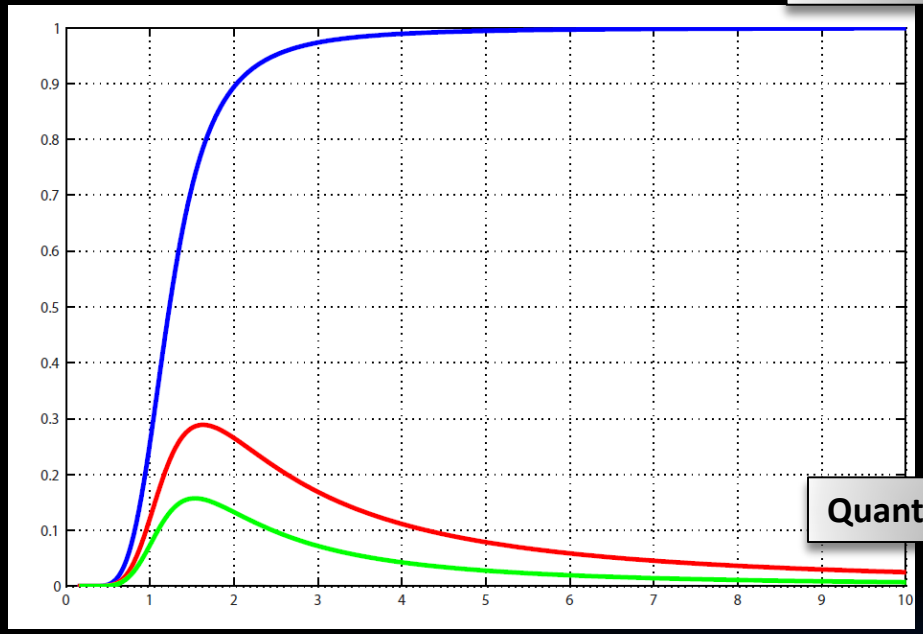


Quantum



Composite System with 10 Perturbed Coins

Spin Chain at Temperature T



Suen W.Y., M. Gu, J. Thompson, V.Vedral. *in Prepatation*.

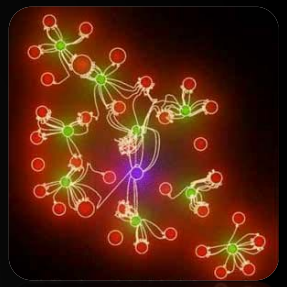
Conclusions

Occam's Razor is Sharper When Quantized.

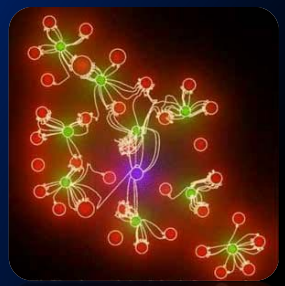
Quantum Theory allow us to build models of the same statistical phenomena while storing less extraneous past information than any classical alternative – even if we are modelling purely classical data.

A Quantum Measure of Complexity

The quantum analogue of the statistical complexity may better align with our intuition of what is complex, and is a very promising measures of complexity for statistical phenomena.

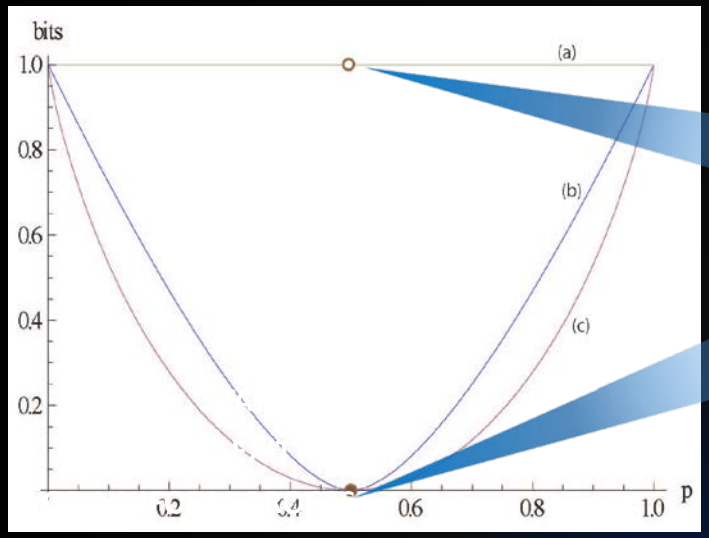


Outlook



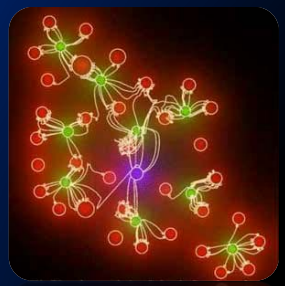
How Different Are Quantum Models?

We know that quantum is simpler – but how much simpler and what significant qualitative differences are there? Can these differences give us a better understanding of complexity or the inherent benefits of quantum information processing?



Classical Statistical Complexity can be very sensitive to noise – and thus its practical evaluation is sometimes tricky. Can quantum overcome this?

Outlook



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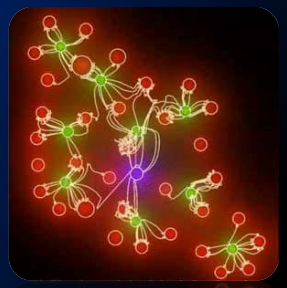
How much simpler can something look Quantum mechanically? Are there Realistic systems where the two measure scale very differently?

Outlook



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Thermodynamic Benefits of Quantum Prediction

Landauer's erasure principle states that it costs energy to erase information. Classically, this leads to a fundamental energy cost for stochastic simulation. Can quantum simulators break this bound?

PROCEEDINGS OF THE ROYAL SOCIETY A

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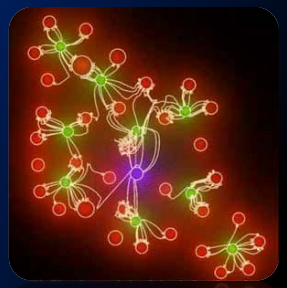
Proc. R. Soc. A (2012) **468**, 4058–4066
doi:10.1098/rspa.2012.0173
Published online 3 October 2012

Information-theoretic lower bound on energy cost of stochastic computation

By Karoline Wiesner[1,*], Mile Gu[2], Elisabeth Rieper[2] and Vlatko Vedral[2,3,4]

Outlook



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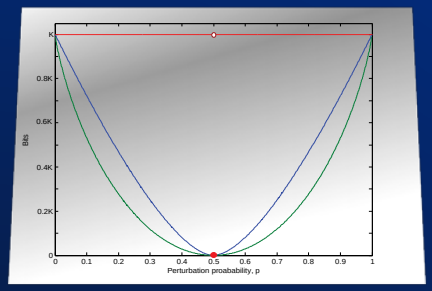
Griffith UNIVERSITY

Experimental Realization

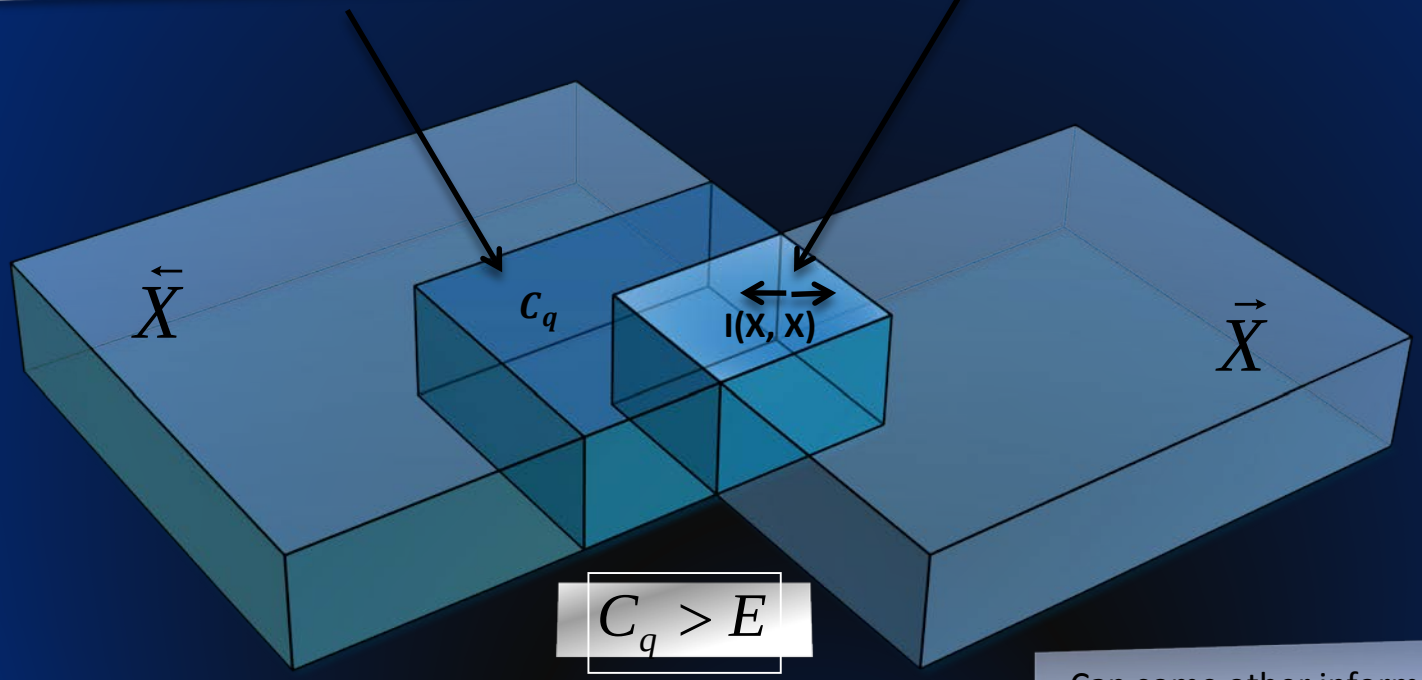
Can we demonstrate quantum simulators in the lab that demonstrate reduced entropy and energetic costs?

Possibilities....



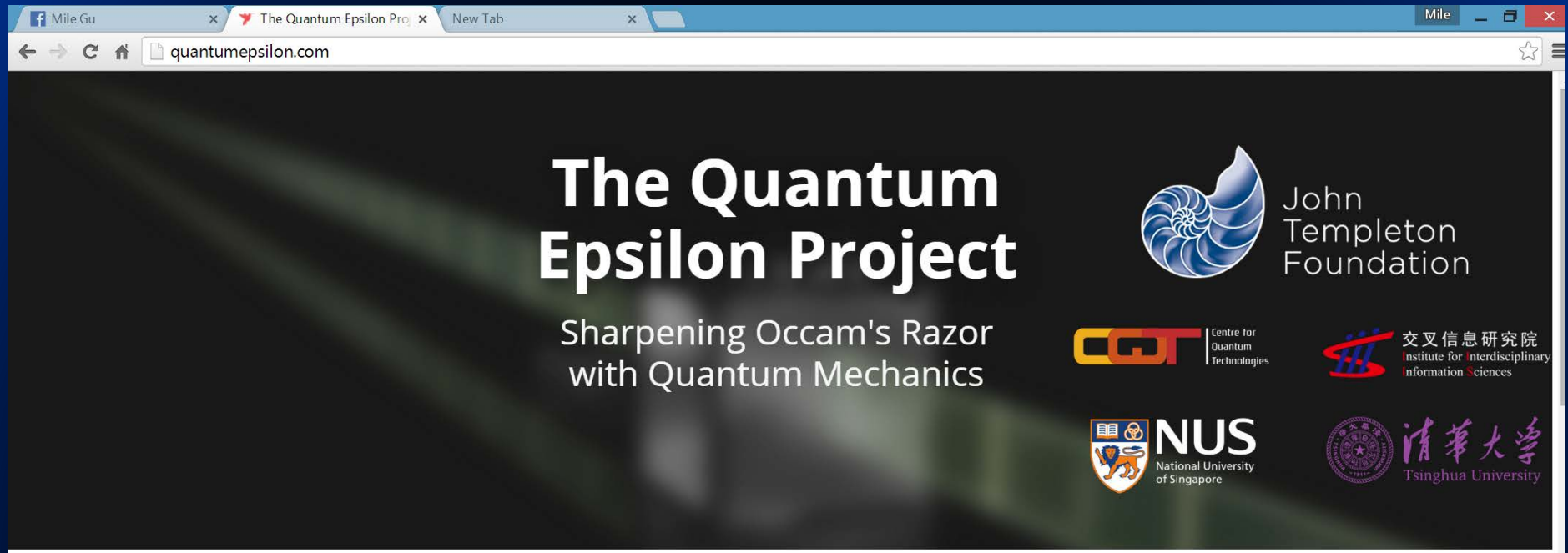
Even quantum models seem require postulating extraneous causes

The amount of useful information the past contains about the future.



Is this a source of causal asymmetry in quantum theory?

Can some other information theory onto quantum theory, or can quantum theory be motivated entirely by Occam's Razor?



Reality's quantum nature is its most inexplicable feature. The outcome of every observation we make can ultimately be written on classical pieces of paper. Why would understanding this classical data merit non-classical logic? This puzzle has pushed a heated search for fundamental physical principles to justify why reality is quantum mechanical.

The Quantum Epsilon Project has been formed through the auspices of the John Templeton Foundation to explain this paradox. We seek to isolate quantum theory from philosophical principles, that arise from a novel interplay of the foundational ideas in computational mechanics and quantum theory. The former to capture the ideal of *Occam's Razor* – the preference for understanding reality through the least extraneous causes. The latter to understand exactly how this notion of 'least extraneous causes' depends on what sort of information theory we use. Together these concepts suggest an intriguing new line of inquiry: could the desire for simplicity isolate quantum theory as the ideal way to understand reality?

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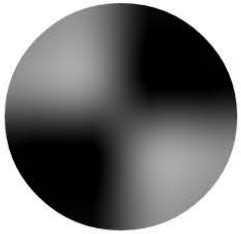
Vlatko Vedral
Project co-leader

Jayne Thompson
Research fellow

Andrew Garner
Research fellow

Students

NAT. UNI. SINGAPORE



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PhD student, CQT

Liu Qing
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TSINGHUA UNIVERSITY

REFERENCES

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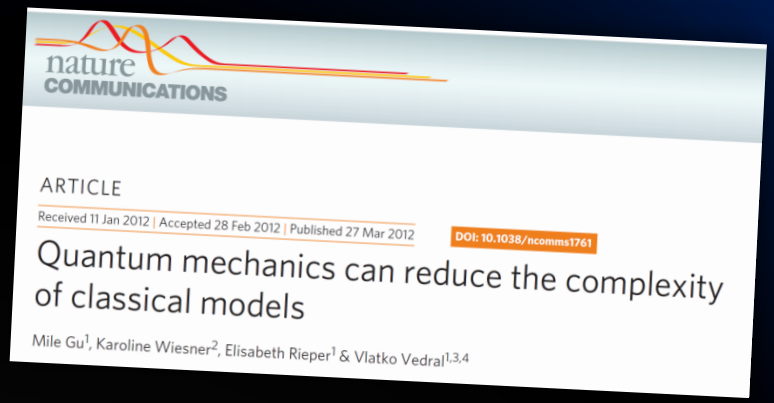
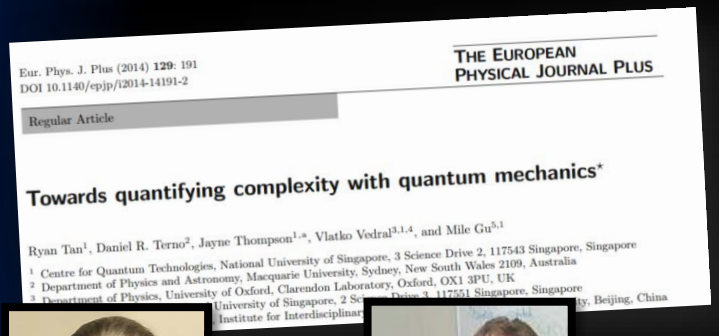
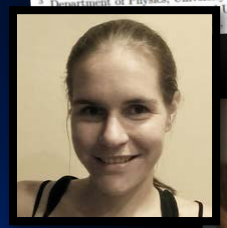


Nature Communications, 3, 762



Ryan Tan



Jayne Thompson

Daniel Terno

Vlatko Vedral

New Scientist, 15/11/2014
pg 28-29