QBism: An Exercise in Thinking Quantum Theory from a New Direction

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"Go it," I almost cried aloud, "and go it *stronger!*"

--- William James on 1906 San Francisco earthquake

Pablo Picasso, *Le Vieux Marc* (oil on canvas), 1912.

QBism puts the scientist back into science

A participatory view of science resolves quantum paradoxes and finds room in classical physics for 'the Now', says **N. David Mermin**.

Physical science describes the objective external world: particles, waves and fields; how they change in time; and how they give rise to the forms of matter, terrestrial and extraterrestrial, microscopic and macroscopic. This world makes itself known to each of us through our own private internal only link with the external world.

In *Nature and the Greeks*[1], Austrian physicist Erwin Schrödinger traced the removal of the subject from science back more than two millennia. Alongside the spectacular success of physical science, this exclusion of personal experience has given rise to some vexing and

that the perceiving subject has as important a role to play in understanding the nature of physical science as does the perceived object.

The first problem is the notorious disagreement, confusion and murkiness that for almost a century has plagued the foundations of quantum mechanics, in spite of the theory's

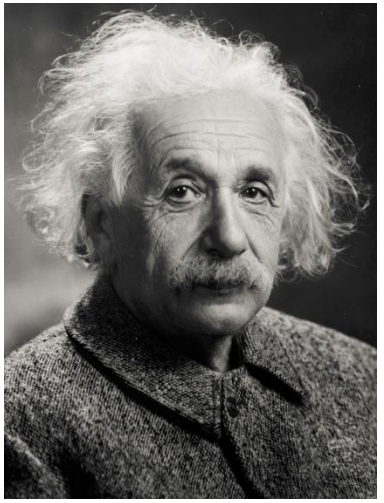
The Milieu of Many Philosophy of Science Meetings:

Nonlocality Rules the Day!

"What Bell proved, and what theoretical physics has not yet properly absorbed, is that the physical world itself is nonlocal."

-- Tim Maudlin
"What Bell Did," 2014

Einstein's Worry



If one asks what is characteristic of the realm of physical ideas independently of the quantum-theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, i.e., ideas are posited of things that claim a "real existence" independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the one hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things "lie in different parts of space." Without such an assumption of the mutually independent existence (the "being-thus") of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation. …

For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the ``principle of local action,'' The complete suspension of this basic principle would make impossible the idea of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.

Maudlin's Worry On "Superdeterminism"

Recall Schrödinger's class of identically prepared students. We are told they can *all* answer *any* of a set of questions correctly, but each can only answer one, and then forgets the answers to the rest. It's an odd idea, but we can still test it: we ask the questions at random, and find that we always get the right answer. Of course it is *possible* that each student only knows the answer to one question, which always happens to be the very one we ask! But that would require a massive coincidence, on a scale that would undercut the whole scientific method. Or else we are being *manipulated*: somehow we are led to ask a given question only of the rare student who knows the answer. So we switch our method of choice, handing it over to a random number generator, or the throw of dice, or to be determined by the amount of rainfall in Paraguay. But maybe all of *these* have been somehow rigged too! Of course, such a purely abstract proposal cannot be *refuted*, but besides being insane, it too would undercut scientific method. All scientific interpretations of our observations presuppose that they have not have been manipulated in such a way.

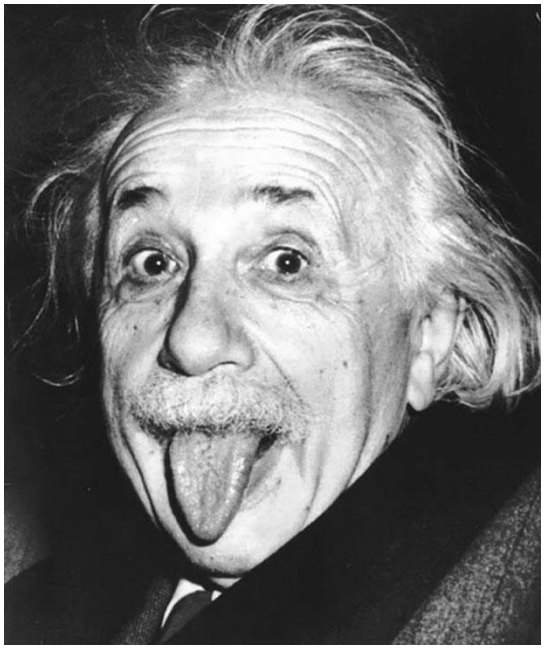
Somehow quantum mechanics hits the sweet spot:

- The world is nonlocal,
- Yet, we can still do science.

Where Does It Stop?

Captain Quantum Entanglement's
Unfortunate Weakness

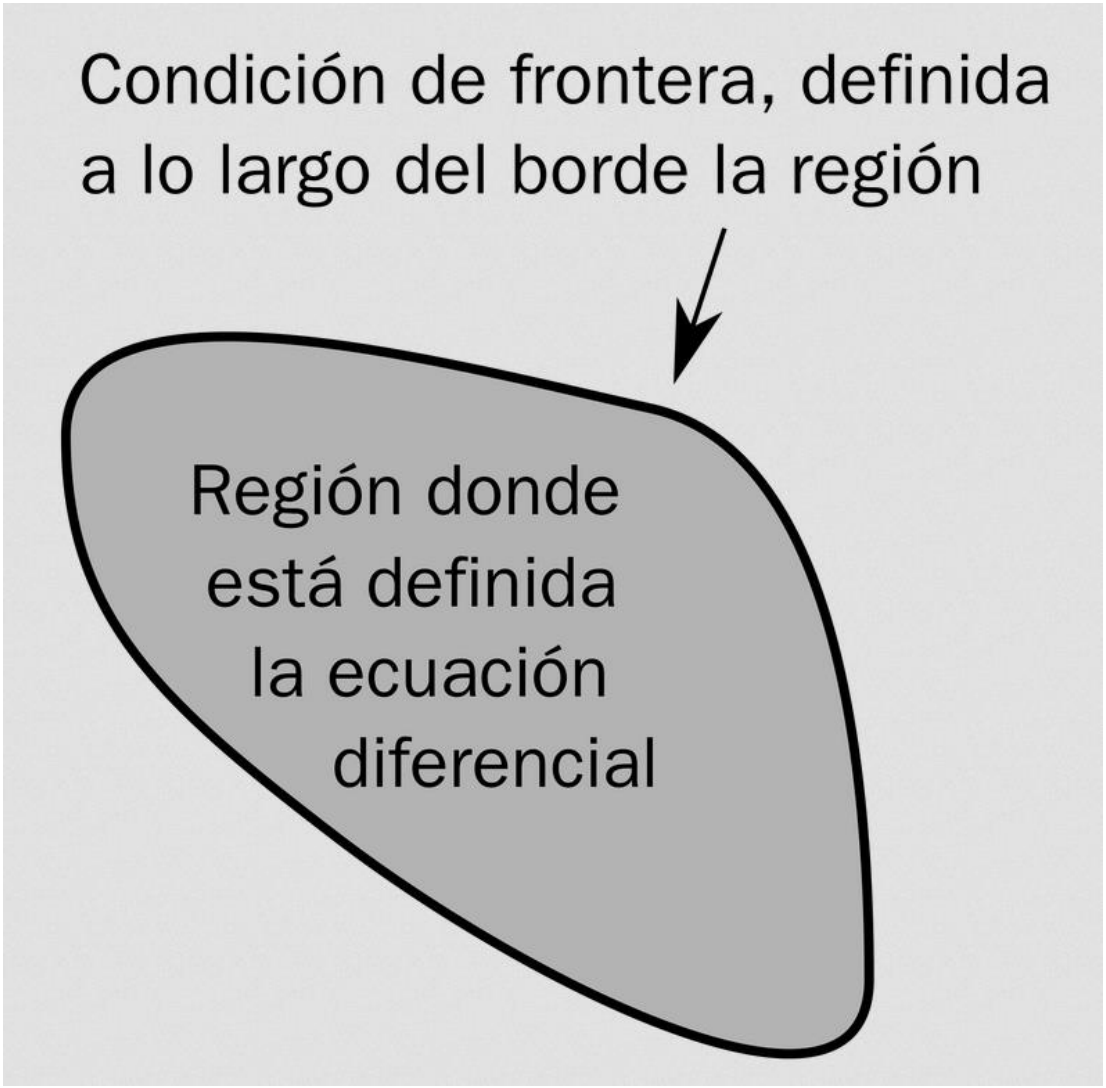
Einstein's Worry, More Detail



… Nor does one see how physical laws could be formulated and tested without such a clean separation. **Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of the one another that it takes as basic, as well as the elementary laws it postulates for them.**

For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no *immediate* effect on B; this is known as the "principle of local action," **which is applied consistently in field theory.** The complete suspension of this basic principle would make impossible the idea of (quasi-) closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.

But is that autonomy enough?



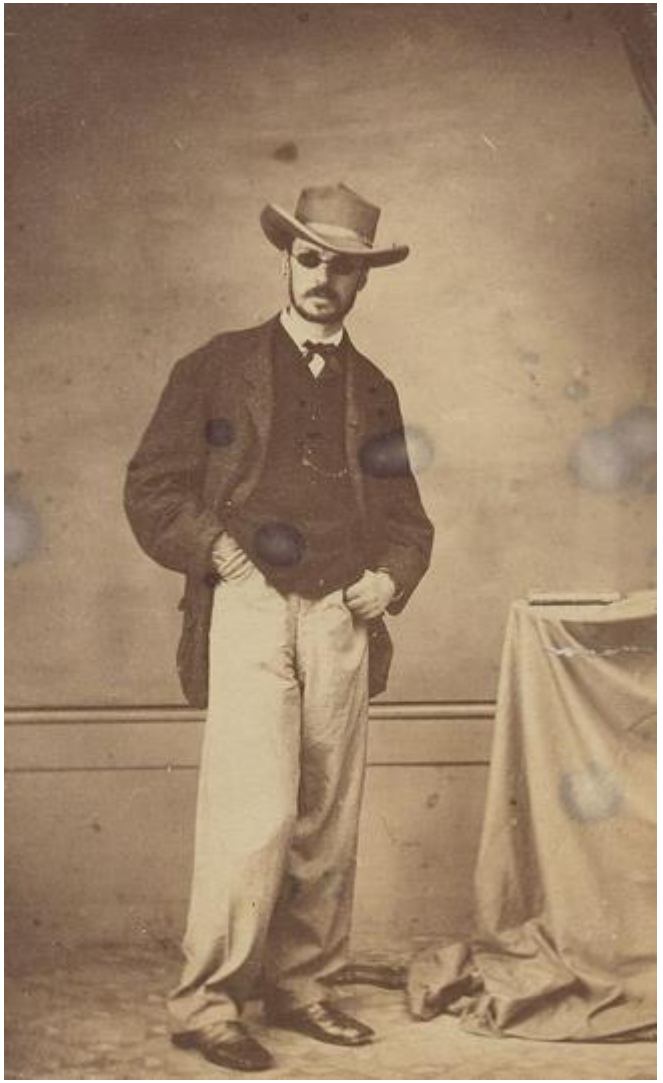
Chris Fields' characterization of QBism (arXiv:1108.2024):

Autonomy all the way down.

Autonomy All the Way Down

Chance is a purely negative and relative term, giving us no information about that of which it is predicated, except that it happens to be disconnected with something else—not controlled, secured, or necessitated by other things in advance of its own actual presence. What I say is that it tells us nothing about what a thing may be in itself to call it "chance." All you mean by calling it "chance" is that this is not guaranteed, that it may also fall out otherwise. For the system of other things has no positive hold on the chance-thing. Its origin is in a certain fashion negative: it escapes, and says, "Hands off!" . . . coming, when it comes, as a free gift, or not at all.

This negativeness, however, and this opacity of the chance-thing when thus considered *ab extra*, or from the point of view of previous things or distant things, do not preclude its having any amount of positiveness and luminosity from within, and at its own place and moment. All that its chance-character asserts about it is that there is something in it really of its own, something that is not the unconditional property of the whole. If the whole wants this property, the whole must wait till it can get it. That the universe may actually be a sort of joint-stock society of this sort, in which the sharers have both limited liabilities and limited powers, is of course a simple and conceivable notion.



William James, 1842 -1910

FRANKFURTER ALLGEMEINE SONNTAGSZEITUNG, 9. FEBRUAR 2014, NR. 6

So liegt denn alles im Auge des Betrachters

Quantenphysik ist absurd. Warum beschreibt sie dann die Welt so exakt wie nichts sonst? Nach Jahrzehnten der Debatte liegt nun ein Vorschlag vor, der das Rätsel vielleicht lösen kann.

VON ULF VON RAUCHHAUPT

„Es ist falsch zu glauben, die Aufgabe der Physik sei es, etwas über die Natur herauszufinden. Physik befasst sich damit, was wir über die Natur sagen können." Dieser Satz steht in einem Nachruf auf Niels Bohr (1885 bis 1962), den Kopenhagener Erzvater der Quantenphysik – die Theorie, durch die Atome, Licht und Elementarteilchen erst exakt beschreibbar wurden. Und nun soll diese Beschreibung nicht der Natur selbst gelten, sondern nur unserem Wissen darüber? Tatsächlich hat der Verfasser der Denkschrift, Bohrs langjähriger Assistent Aage Petersen, die Position des Meisters akkurat beschrieben – und die ist eigentlich ein Skandal.

Physikphilosophen hadern damit bis heute. Der Vorwurf lautet auf Verrat am Ideal eines rationalen Weltbildes, auf Mystizismus oder zumindest Beihilfe dazu. Der Abwehrkampf gegen die „Kopenhagener Interpretation" der Quantentheorie tobt schon Jahrzehnte und wird dadurch verschärft, dass es dabei um die am besten durch Beobachtungen gestützte Theorie geht, die es je gab. Warum passt sie dann so gar nicht zu der klassischen Physik, die Planeten, Dampfkessel und Elektromotoren regiert, sondern zeigt nachgerade absurde Züge, etwa Teilchen, die durch zwei Spalten zugleich fliegen? Manche Autoren sind bereit, die Absurditäten lieber zu verlagern und beispielsweise an eine sich ständig in Parallelwelten aufspaltende Wirklichkeit zu glauben, als den quantenphysikalischen Entitäten einen objektiven Charakter abzusprechen.

Seit einigen Jahren gibt es eine neue Interpretation, die vielleicht in der Frage weiterhelfen kann, wie Quantentheorie zu verstehen ist. Sie nennt sich „QBismus", was passenderweise genauso ausgesprochen wird wie die revolutionäre Stilrichtung der modernen Malerei, sich aber von „Quanten-Bayesianismus" ableitet. Bayesianismus wiederum nennt sich eine Auffassung darüber, was mit dem Wort „wahrscheinlich" gemeint ist (siehe „Was ist Wahrscheinlichkeit?"). Diese Frage betrifft die Quantenphysik schon deswegen, weil deren Formeln im Allgemeinen Wahrscheinlichkeiten ausspucken: Anders als in der klassischen Physik, wo man mit Gleichungen den Ort etwa eines Himmelskörpers zu einem zukünftigen Zeitpunkt berechnen kann, vermag ein Quantenphysiker für den Ort eines Elektrons nur eine Wahrscheinlichkeitsverteilung vorauszusagen. Das bedeu-

Im Unterschied zu den QBisten *erschaffen Kubisten ihre Dinge selber. Juan Gris malte „Gitarre und Klarinette" im Jahr 1920.*

Abbildung Artothek

andere Eigenschaften der Quantenphysik fanden die QBisten Entsprechungen in einer konsequent bayesianisch interpretierten Wahrscheinlichkeitstheorie. Denn hier wie dort geht es um den Grad persön-

lichkeit eine Frau war, denn etwa die Hälfte aller Menschen sind die Frauen. Das ist in diesem Moment für mich der „Zustand" des Geschlechts des Witzboldes.

Nun bekomme ich eine weitere Infor-

Männer. Was aber, wenn die neue Information darin besteht, dass die Person sehr wohl bärtig war? Dann werde ich wohl schlagartig sicher sein, dass es ein Mann war.

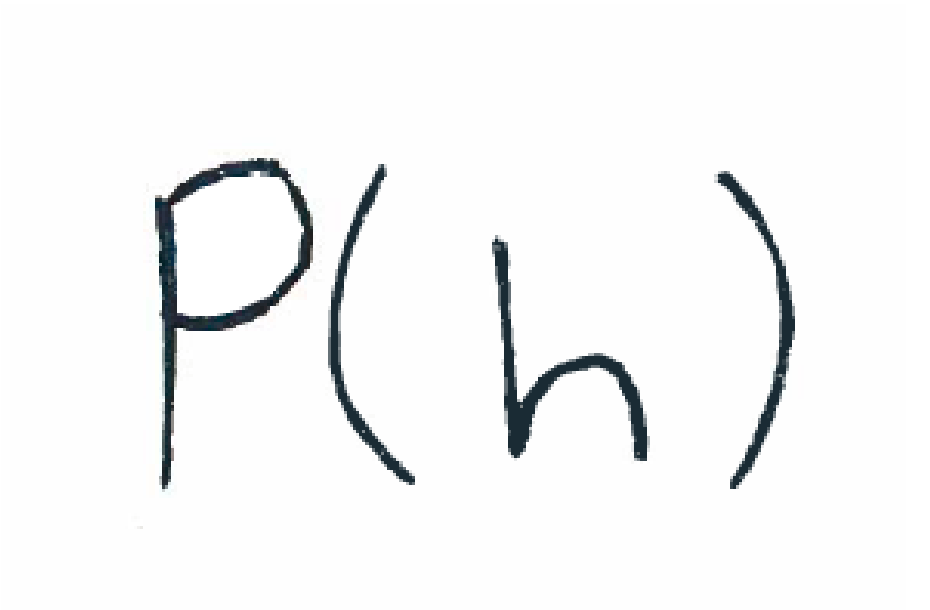
wissenschaftliche Realität für verschiedene Subjekte unterschiedlich. „Das ist nicht so seltsam, wie es klingt", erklärt Fuchs. „Was für einen Akteur real ist, das beruht allein darauf, was dieser Akteur für

Haarig: das Bayes-Theorem

Der schottische Pfarrer Thomas Bayes (1701 bis 1761) befasste sich nebenbei mit Mathematik und insbesondere mit dem Pro-

QBism?

What are Quantum Probabilities? Indeed, what are probabilities?



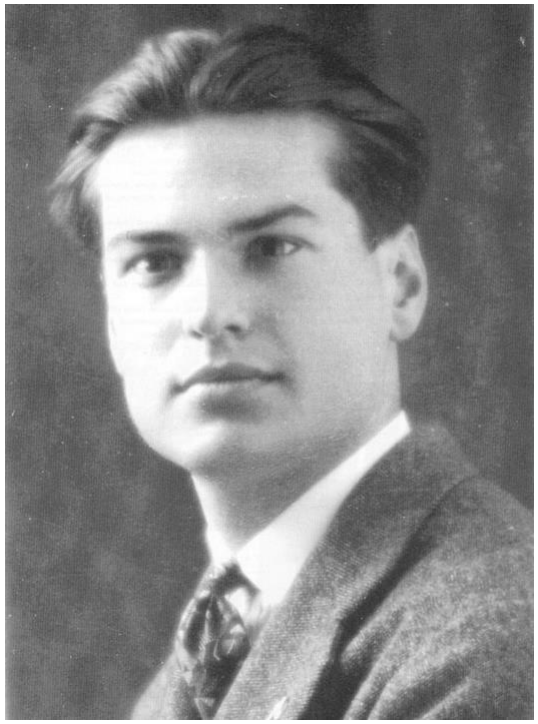
What Are Quantum Probabilities?

- Unless we want tickle, tickle, tickle, they have to be banished from the external world.

My thesis, paradoxically, and a little provocatively, but nonetheless genuinely, is simply this:

PROBABILITY DOES NOT EXIST.

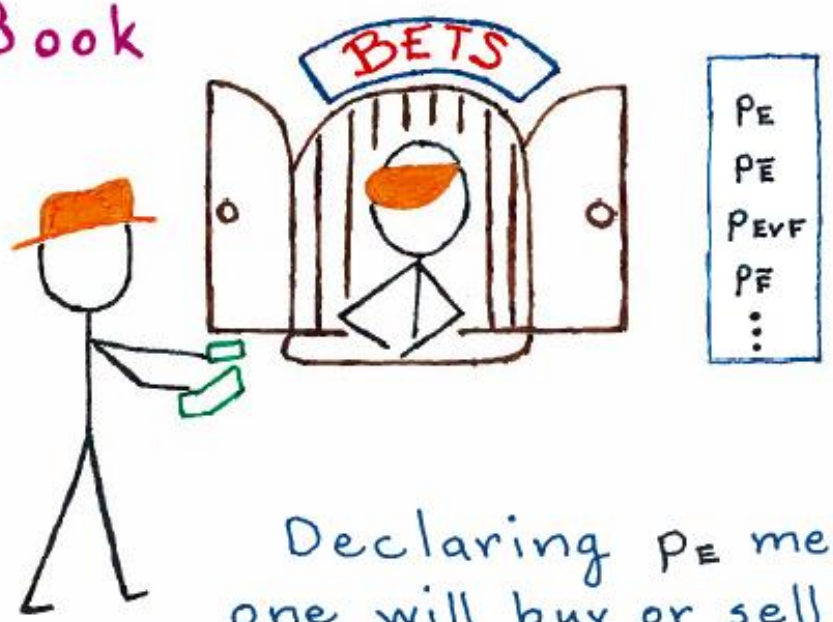
The abandonment of superstitious beliefs about the existence of Phlogiston, the Cosmic Ether, Absolute Space and Time, ..., or Fairies and Witches, was an essential step along the road to scientific thinking. Probability, too, if regarded as something endowed with some kind of objective existence, is no less a misleading conception, an illusory attempt to exteriorize or materialize our actual probabilistic beliefs.



Bruno de Finetti
1906 -- 1985

Defining Probability

Dutch Book



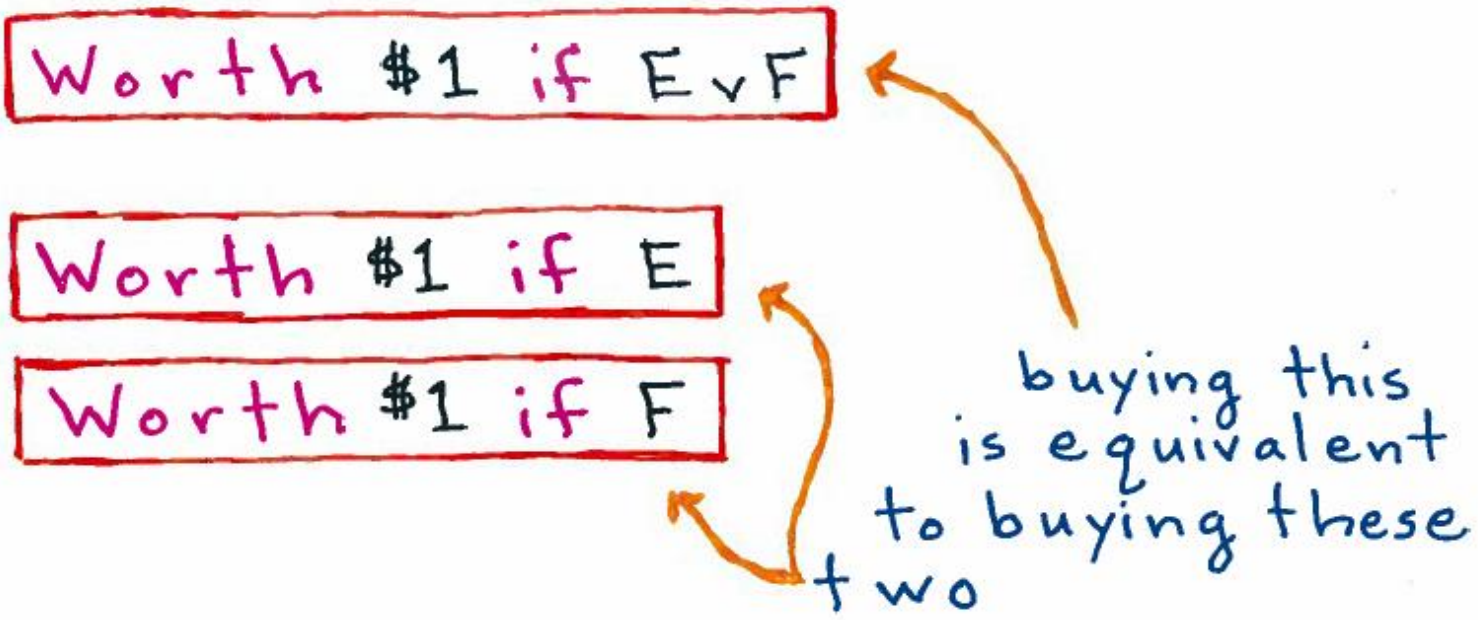
Declaring p_E means one will buy or sell a lottery ticket

Worth \$1 if E

for $\$p_E$.

Example 3:

Suppose E and F mutually exclusive.



So must have $P_{E \vee F} = P_E + P_F$.

A Single-User Theory

- probability theory
- quantum theory

"The Bayesian, subjectivist, or coherent, paradigm is egocentric. It is a tale of one person contemplating the world and not wishing to be stupid (technically incoherent). He realizes that to do this his statements of uncertainties must be probabilistic."

— D. V. Lindley

Why Normative ?
Why Not Necessitarian?

$p(A)$, $p(B)$, $p(C)$, ...

$p(A \mid B)$, $p(A \mid B \vee C)$,

$p(A \wedge B \mid C \vee D)$, $p(A, B)$

$p(A \wedge (B \vee C) \mid Z \wedge Y \wedge (X \vee W))$

$p(Z \wedge (A \vee (L \wedge K)) \mid \neg M \wedge (\neg F \vee Y))$

⋮

No Commitment to Ontology Here

Most of the time one sees Bayesian probabilities characterized as measures of ignorance or imperfect knowledge. But that description carries with it a metaphysical commitment that is not necessary for the personalist Bayesian.

Imperfect knowledge? It sounds like something that, at least in imagination, could be perfected, making all probabilities zero or one—one uses probabilities only because one does not know the true, pre-existing state of affairs.

All that matters is that there is *uncertainty* for whatever reason. There might be uncertainty because there is ignorance of a true state of affairs, but there might be uncertainty because the world itself does not yet know what it will give—i.e., there is an objective indeterminism.

Certainty

What means probability 1?

It means one will buy or sell
a lottery ticket

Worth #1 if E

for in fact #1, full stop.

That is all it means.

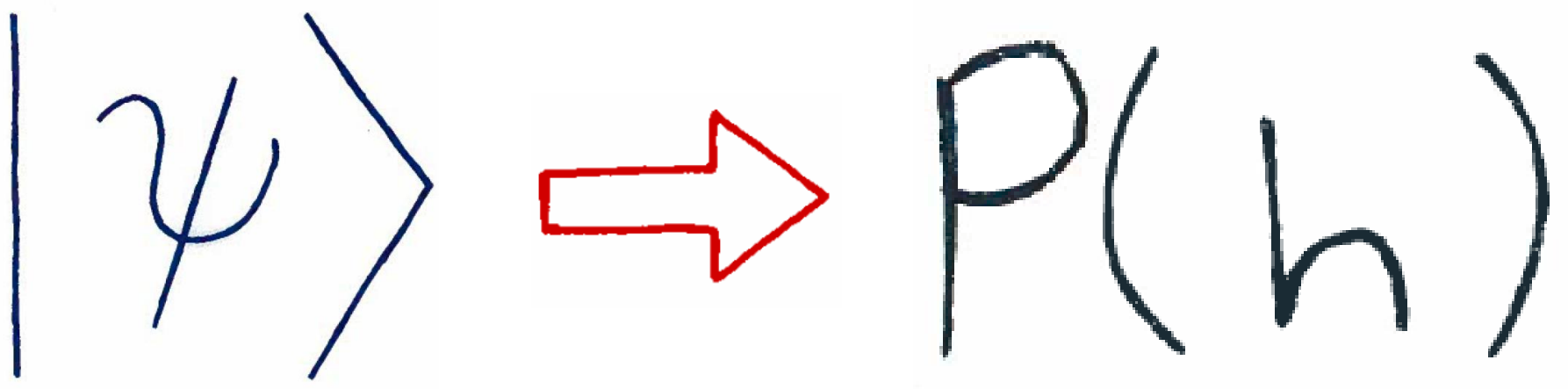
Probability 1 $\not\Rightarrow$ { truth; pre-existent truth; isolated truth }

Certainty

"Certainty is as it were a tone of voice in which one declares how things are, but one does not infer from the tone of voice that one is justified."

– L. Wittgenstein

What We Do with Quantum States



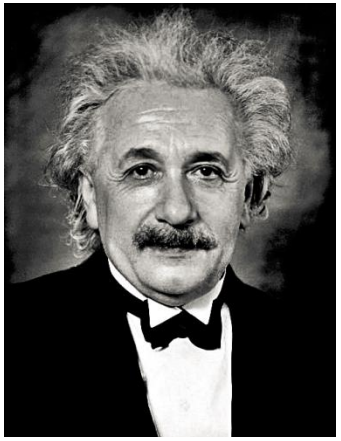
"Measurement"

Does it reveal a pre-existing, but unknown, value?

or

Does it in some sense go toward creating the very value?

Einstein, Podolsky, and Rosen (EPR) Criterion of REALITY



(1935)

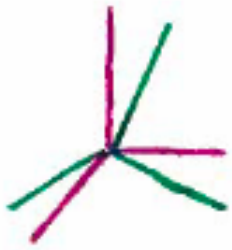
"If, without in any way disturbing a system [one can gather the information required to] predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Modern-Day Version of EPR

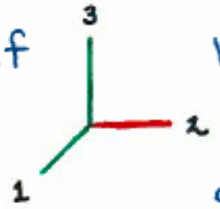
Consider two spatially separated qutrits in a maximally entangled state:

$$|EPR\rangle = \sum_{i=1}^{3} |i\rangle|i\rangle$$

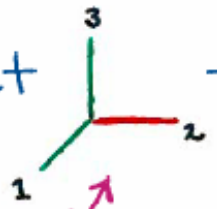
Now measure the left one any way you like. Say with A or B:



So measurement is simple revelation after all?

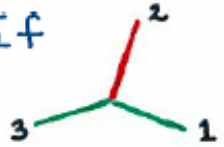
If 

here,

can predict 

there.

element of reality

If 

here,

can predict 

there.

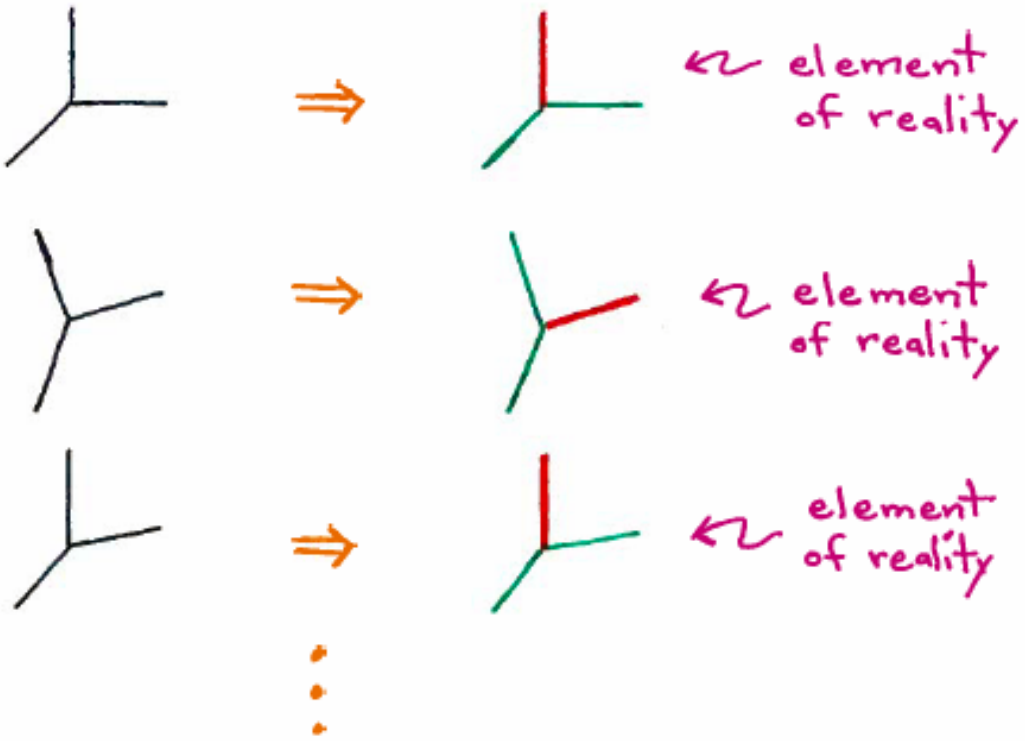
element of reality

Key Argument

Then we should be able to color every set of orthogonal rays in $\mathbb{R}^3$ red-green-green.

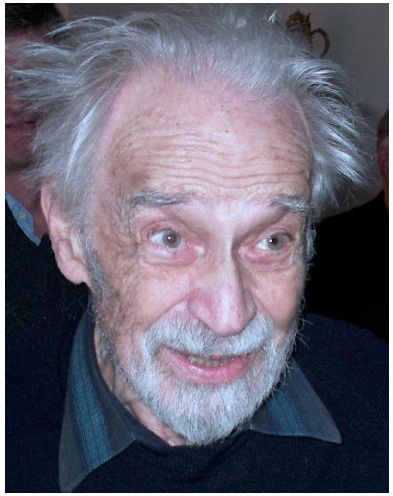
EPR Implodes

But must consider many more bases than two.



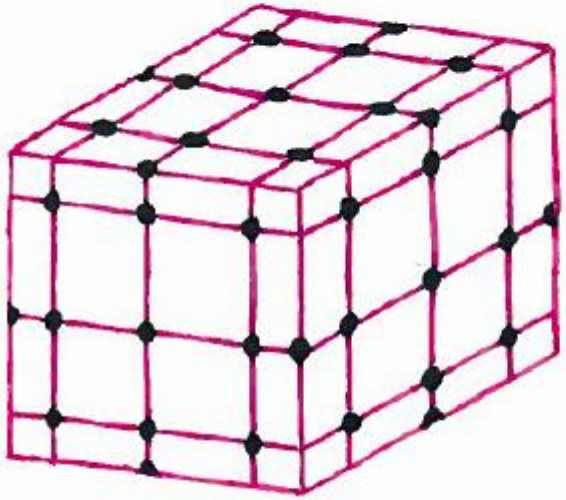
Until contradiction.

Simon Kochen



Ernst Specker
1920 – 2011(?)

Cannot be colored:



33 rays, Peres

(when completed into full triads, consists of 40 triads made from 57 rays)

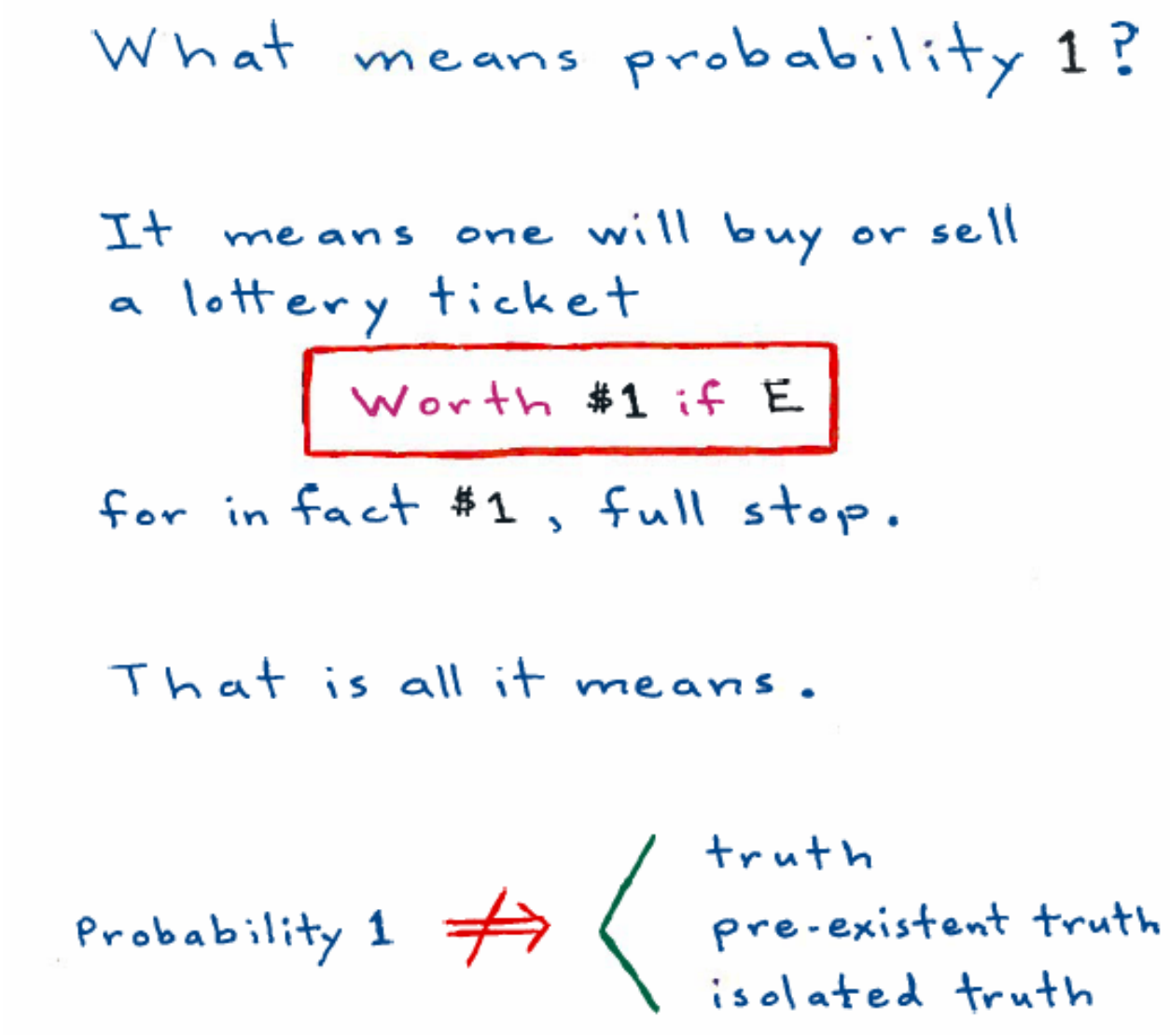
Asher Peres, 1934-2005

Nonlocality, right?

"If I could make one change to the EPR paper in retrospect it would be to alter the characterization of this criterion. The authors call it 'reasonable' and 'in agreement with classical as well as quantum-mechanical ideas of reality', but its status is actually much stronger than that: the criterion is, in the parlance of philosophers, *analytic*. That is, this criterion follows just from the very meanings of the words used in it."

-- Tim Maudlin
"What Bell Did," 2014

But, remember in QBism:



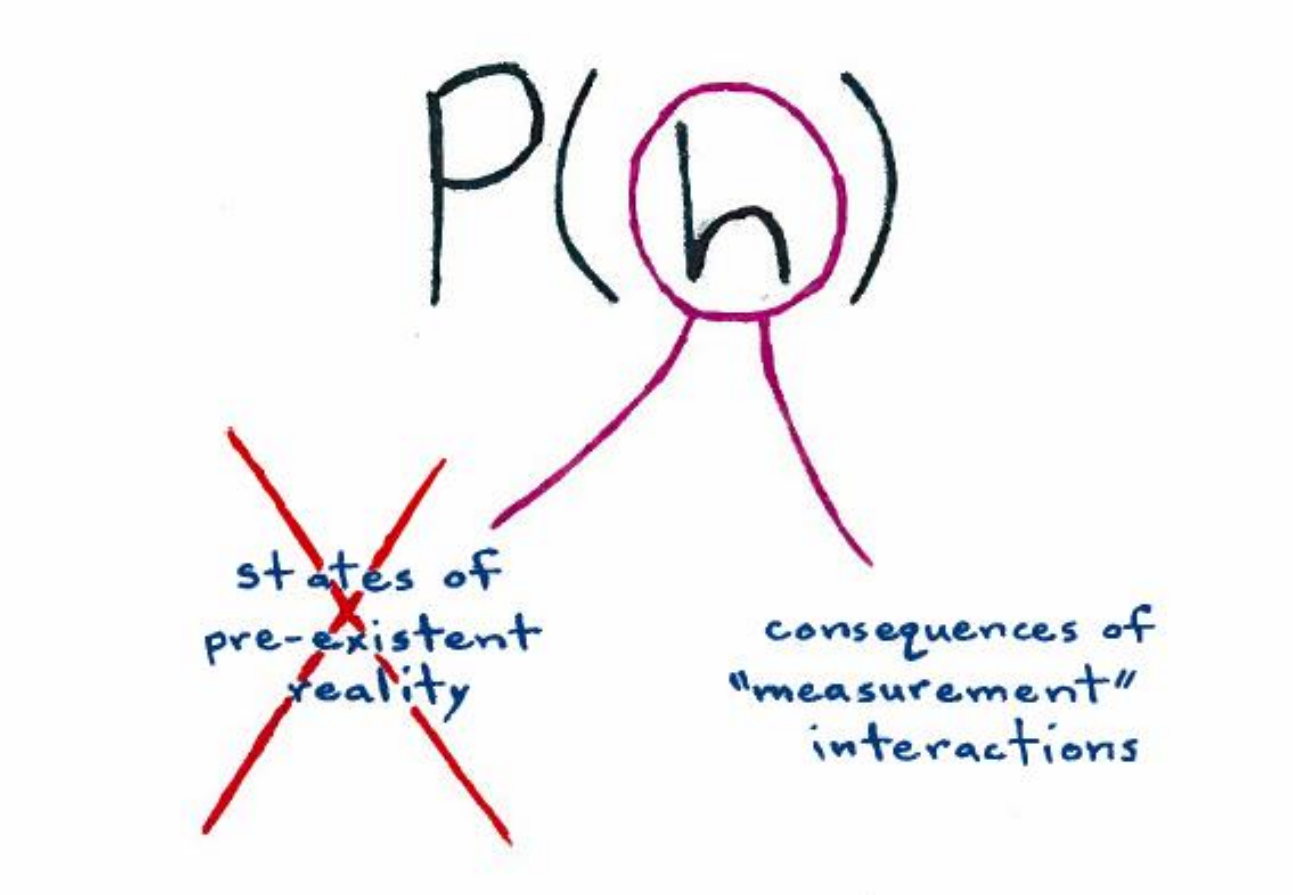
"Measurement"

Does it reveal a pre-existing, but unknown, value?

or

Does it in some sense go toward creating the very value?

What Quantum Probabilities Are About

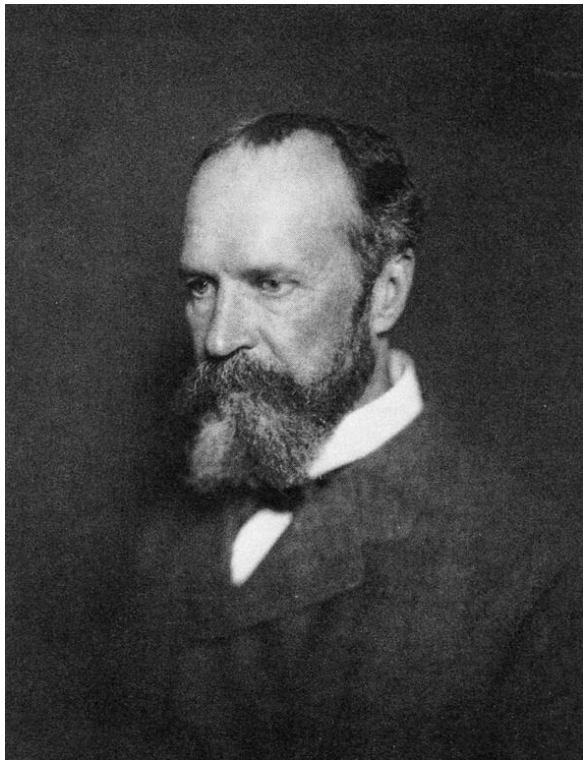


“Our Experience in/on/with the External World”

-- with apologies to Bertrand Russell

In our cognitive as well as in our active life we are creative. We *add*, both to the subject and to the predicate part of reality. The world stands really malleable, waiting to receive its final touches at our hands. Like the kingdom of heaven, it suffers human violence willingly.

— William James

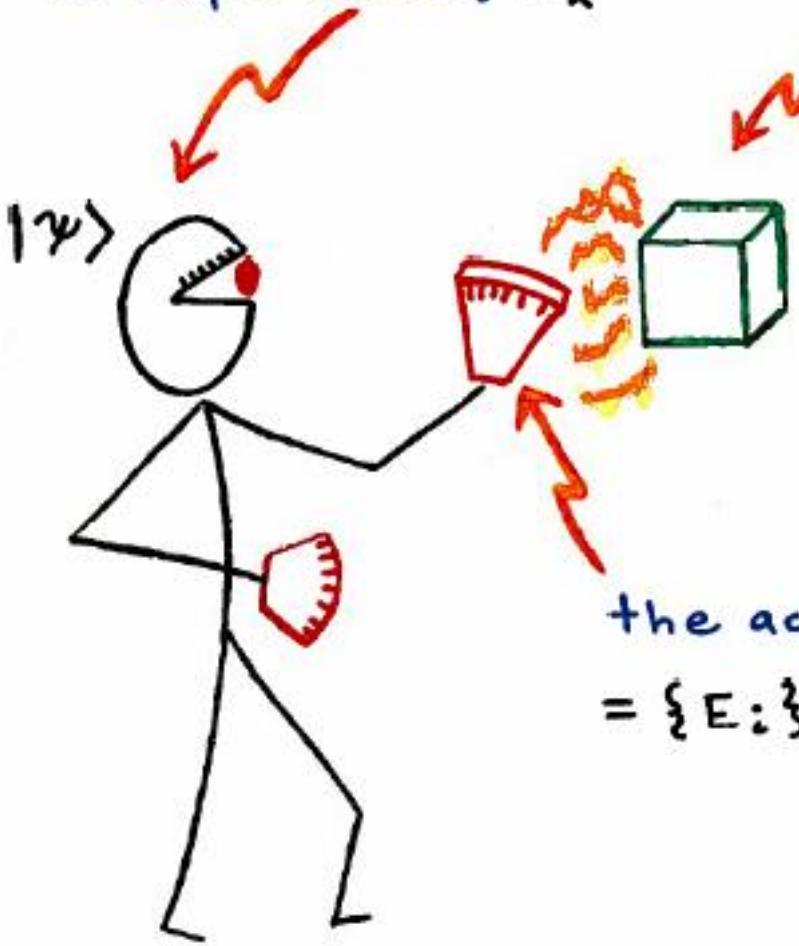


the consequence

= an experience, E_k

the catalyst

= quantum system, ψ_d



the action

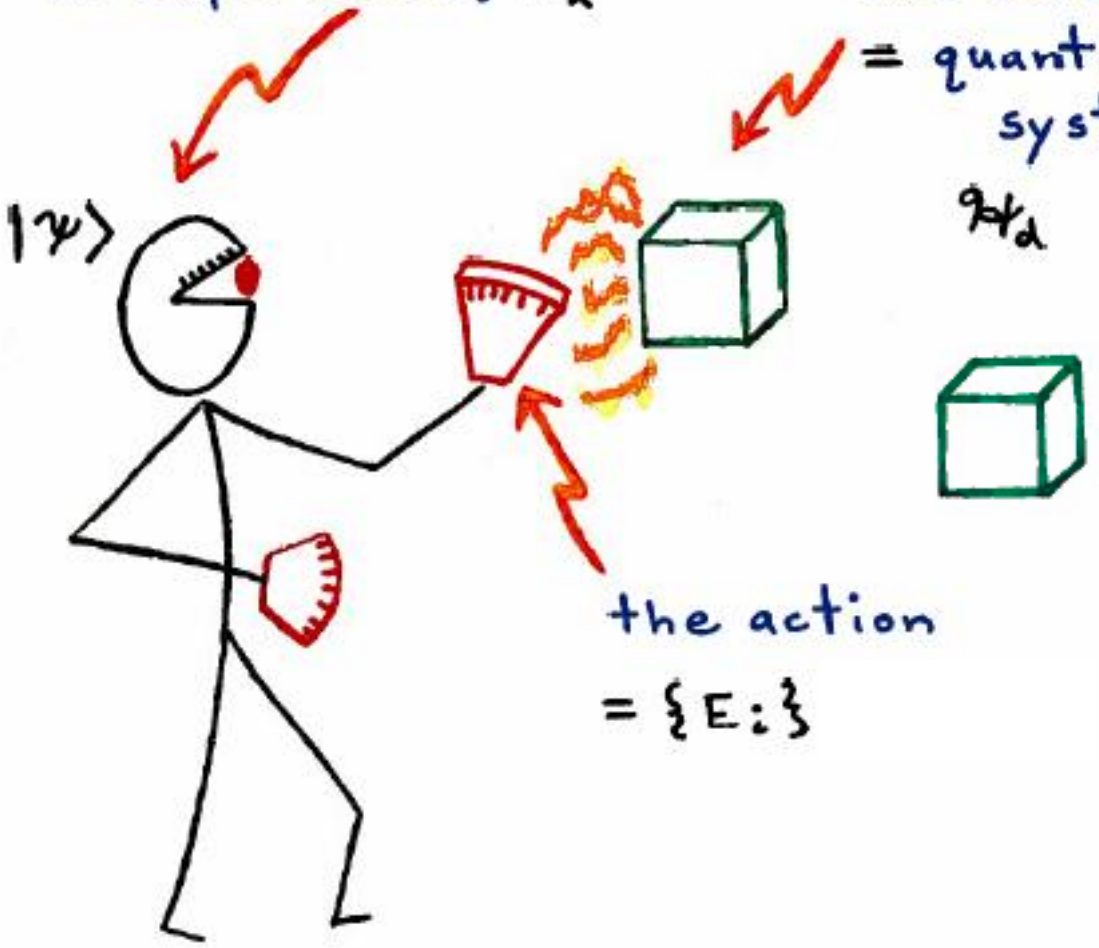
$= \{E_i\}$

the consequence

= an experience, E_k

the catalyst

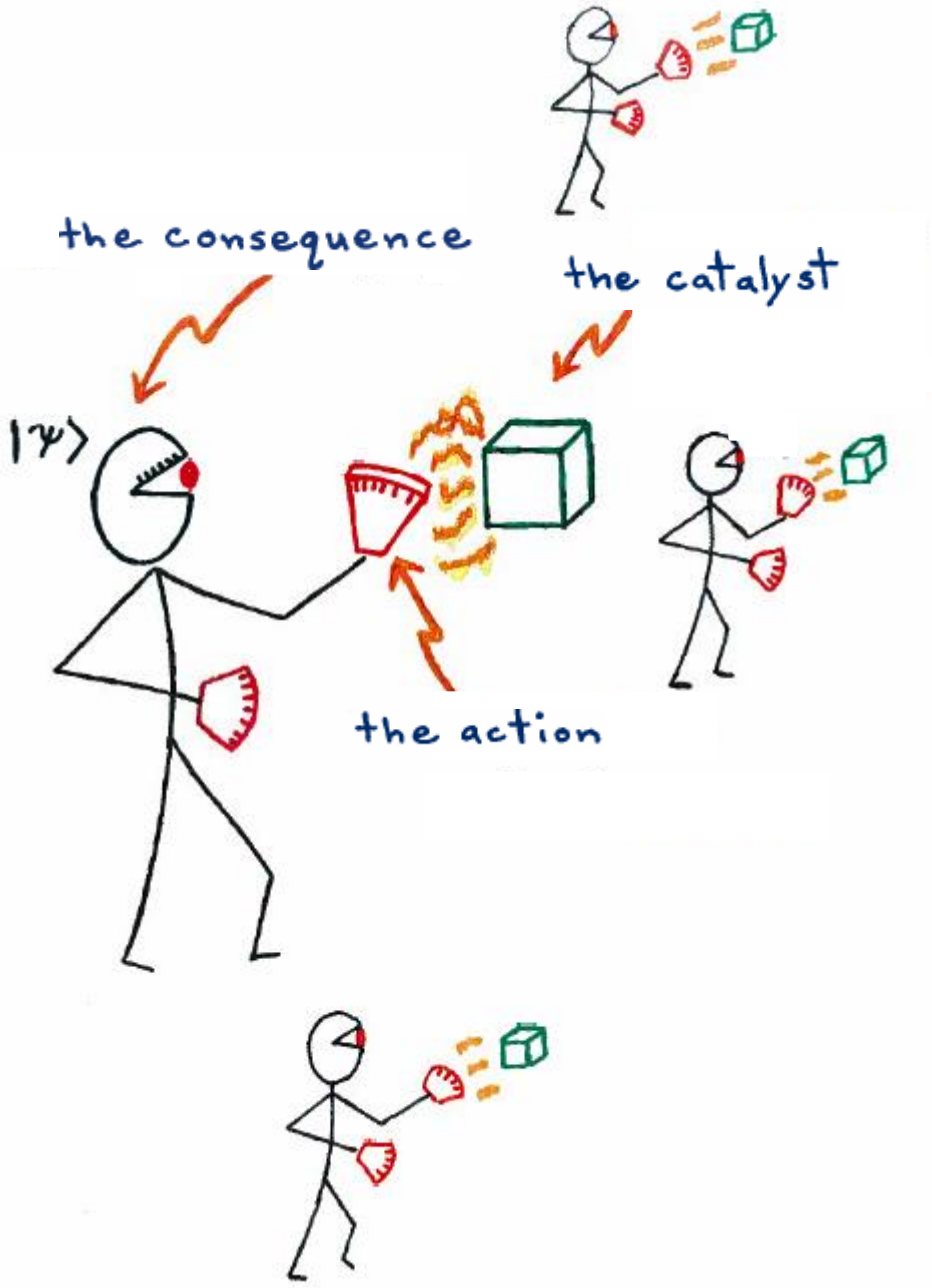
= quantum system, ψ_d



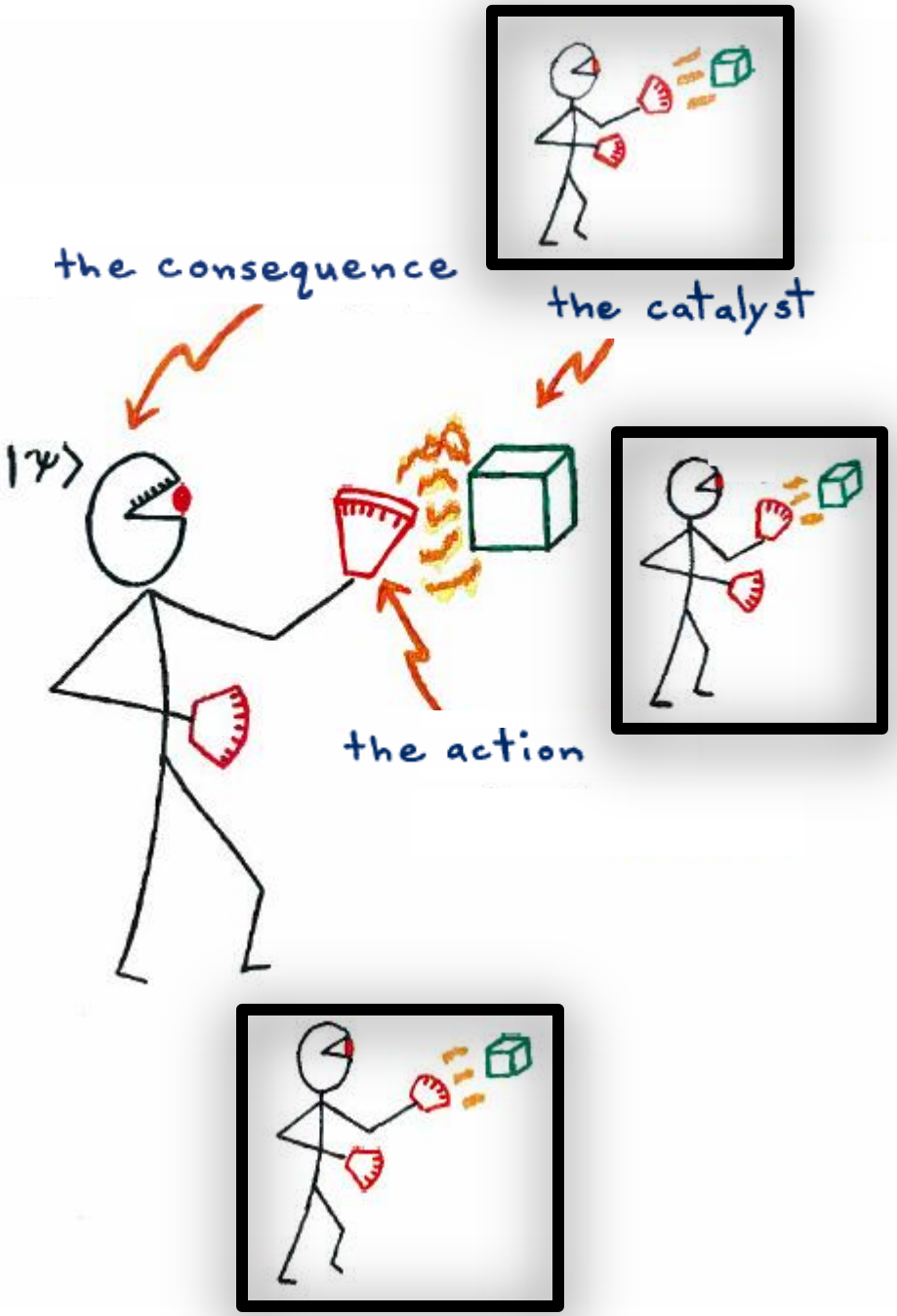
the action

$= \{E_i\}$

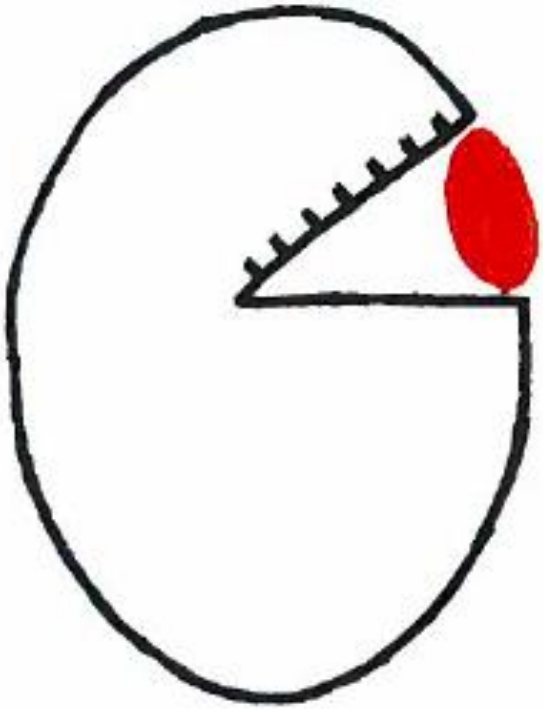
Quantum theory can be used by anyone.



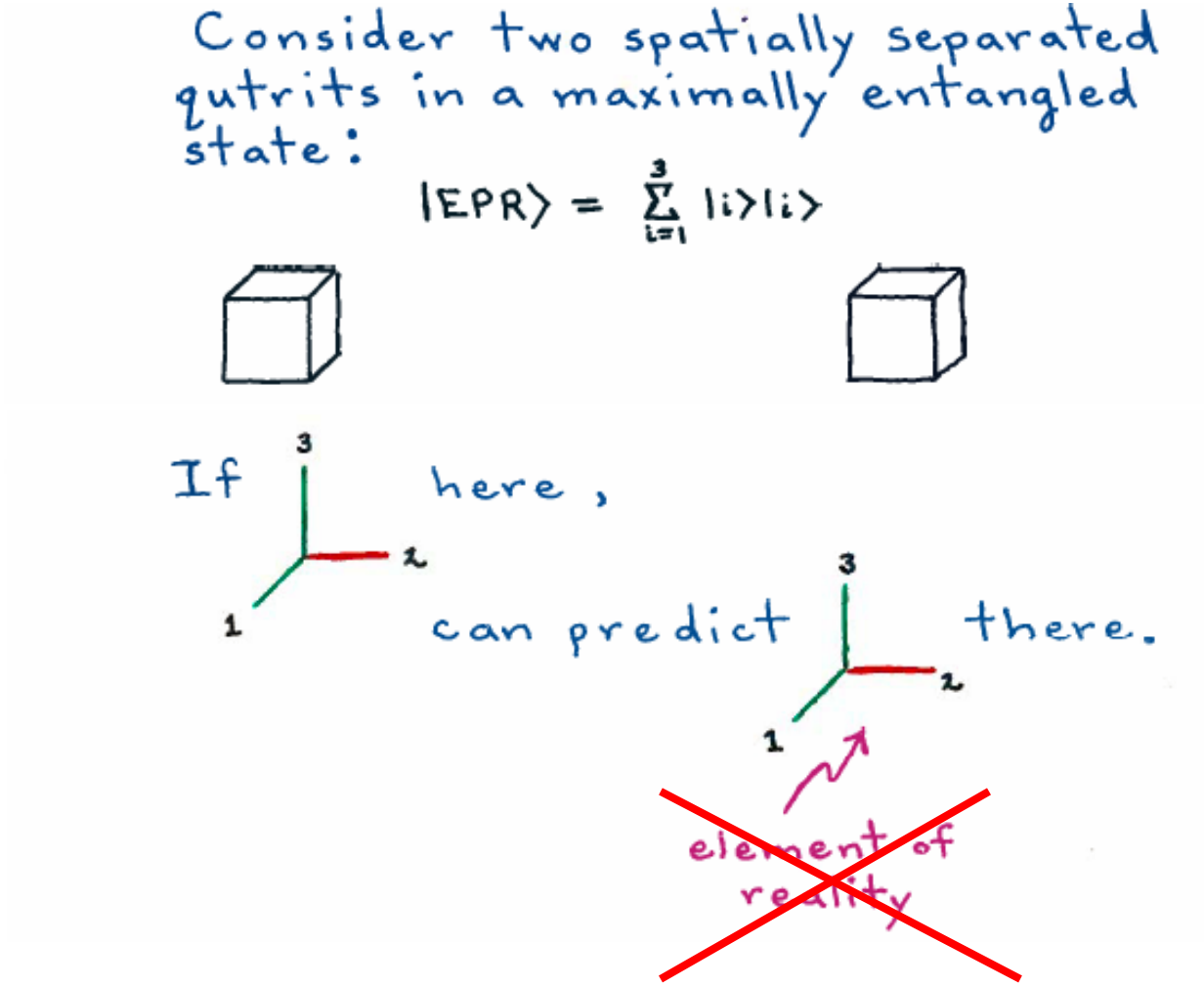
But those other users, for the agent in focus, are physical systems like anything else.



What QBism Is Not



EPR Redux … QBist Style



QBism's Story: Take an action, walk, take an action.

Example 1: The Pure Einstein

Alice measures one half of an EPR pair, updating to $|\psi\rangle$ for the other side.

All it means is if she were to walk to the other side and measure $\{|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|\}$ she would gamble her life on getting outcome $|\psi\rangle\langle\psi|$.

QUANTUM MYSTERIES FOR ANYONE

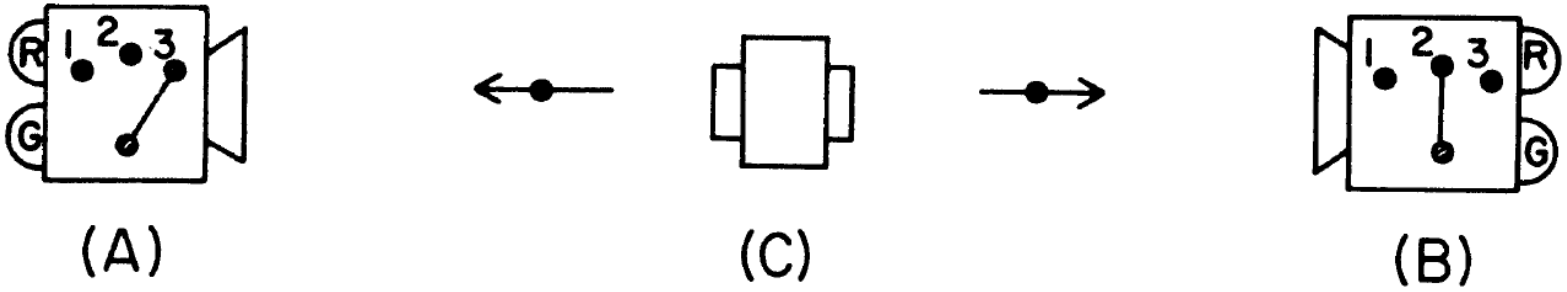


Fig. 2. The complete device. A and B are the two detectors. C is the box from which the two particles emerge.

N. David Mermin

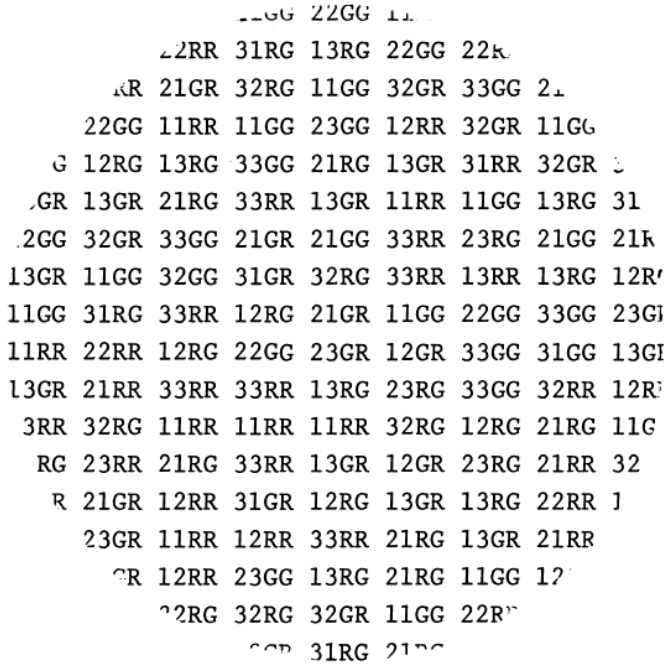

22RR 31RG 13RG 22GG 22R
R 21GR 32RG 11GG 32GR 33GG 2
22GG 11RR 11GG 23GG 12RR 32GR 11G
G 12RG 13RG 33GG 21RG 13GR 31RR 32GR
GR 13GR 21RG 33RR 13GR 11RR 11GG 13RG 31
2GG 32GR 33GG 21GR 21GG 33RR 23RG 21GG 21
13GR 11GG 32GG 31GR 32RG 33RR 13RR 13RG 12R
11GG 31RG 33RR 12RG 21GR 11GG 22GG 33GG 23G
11RR 22RR 12RG 22GG 23GR 12GR 33GG 31GG 13G
13GR 21RR 33RR 33RR 13RG 23RG 33GG 32RR 12R
3RR 32RG 11RR 11RR 11RR 32RG 12RG 21RG 11G
RG 23RR 21RG 33RR 13GR 12GR 23RG 21RR 32
R 21GR 12RR 31GR 12RG 13GR 13RG 22RR
23GR 11RR 12RR 33RR 21RG 13GR 21RR
R 12RR 23GG 13RG 21RG 11GG 1
2RG 32RG 32GR 11GG 22R
31RG

Fig. 4. Fragment of a page of a volume from the set of notebooks recording a long series of runs.

Example 2: Scenario of Bell Inequality Tests

Alice and Bob set out to demonstrate Bell inequality violations.

Alice believes quantum mechanics.

Alice's beliefs evolve:

initial $|EPR\rangle\langle EPR| \otimes \rho_{Bob} \equiv \rho_0$

believing Bob interacts with his qubit

$\rightarrow (I \otimes U_{QB})\, \rho_0\, (I \otimes U_{QB}^{+})$

she measures, updates Bob & his qubit

$\rightarrow$ ρ_{QB} generally entangled

Example 2 cont.

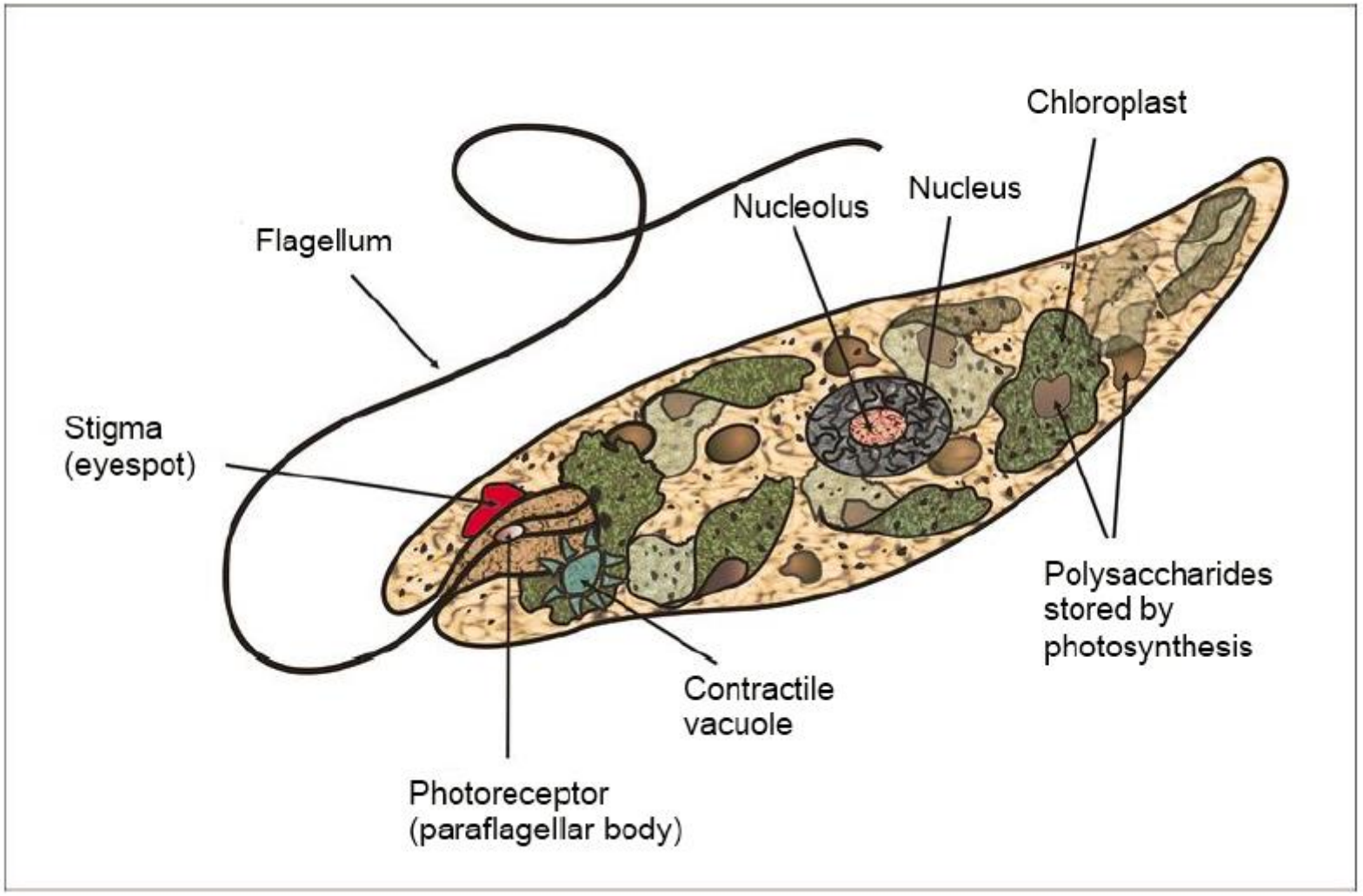
Where in Alice's beliefs (i.e. quantum states) is any notion of clicks on Bob's side?

Her quantum states do not pierce into those systems.

They only refer to what she believes will be the consequences of her later interactions with Bob.

Done, right?

A way to think about QBism ...



Calculus 1  Character 1

Calculus 2  Character 2

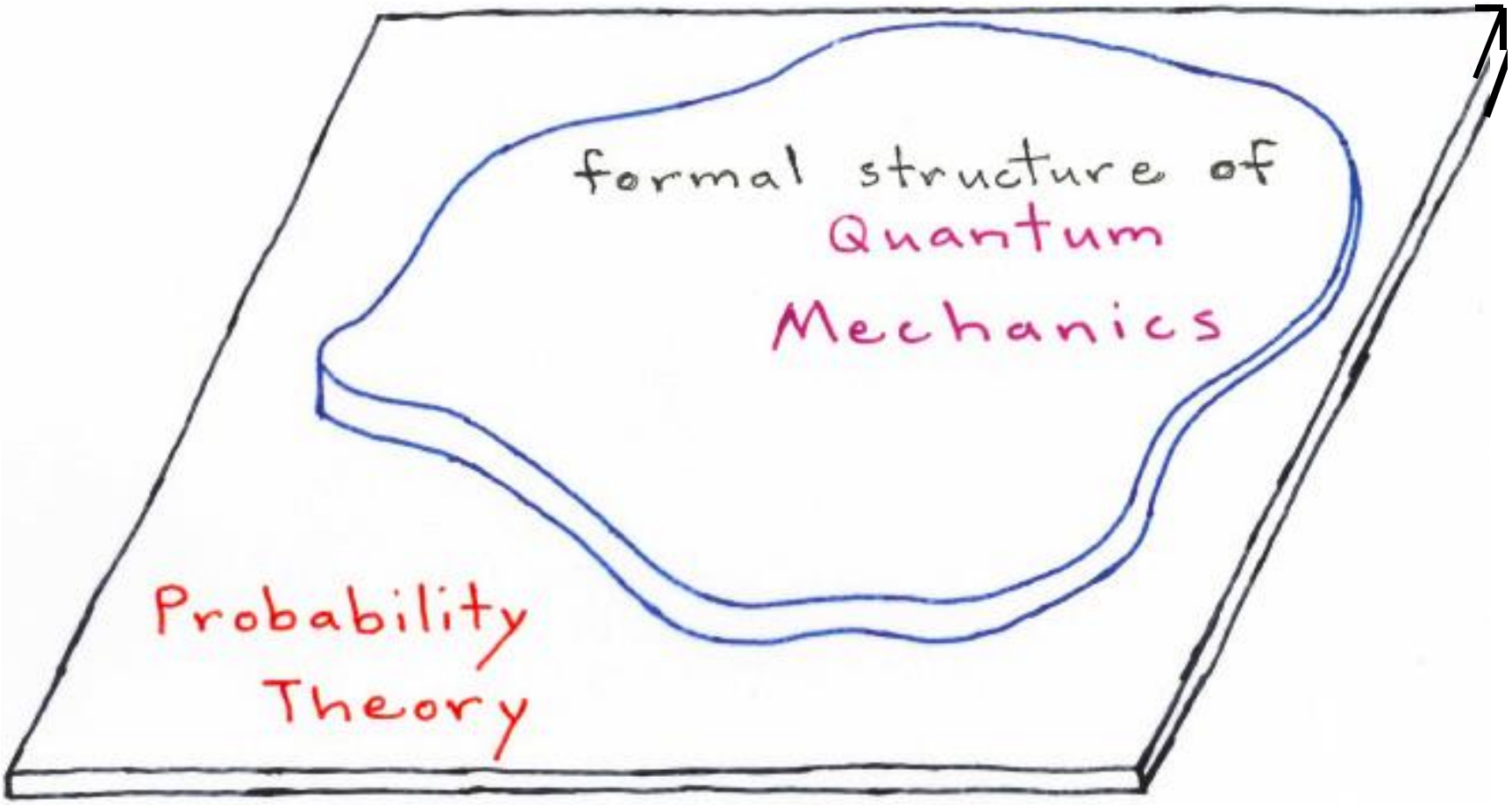
Calculus 3  Character 3

The Born Rule

Given ρ and $\{E_i\}$,

$$p(i) = \mathrm{tr}\, \rho E_i$$

"The Born Rule"


formal structure of
Quantum
Mechanics
Probability
Theory

The Born Rule

Given ρ and $\{E_i\}$,

$$p(i) = \mathrm{tr}\,\rho E_i$$

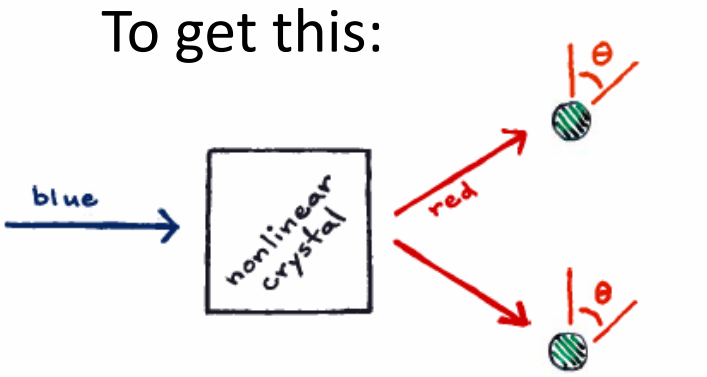
"The Born Rule"

NOT a law of nature.

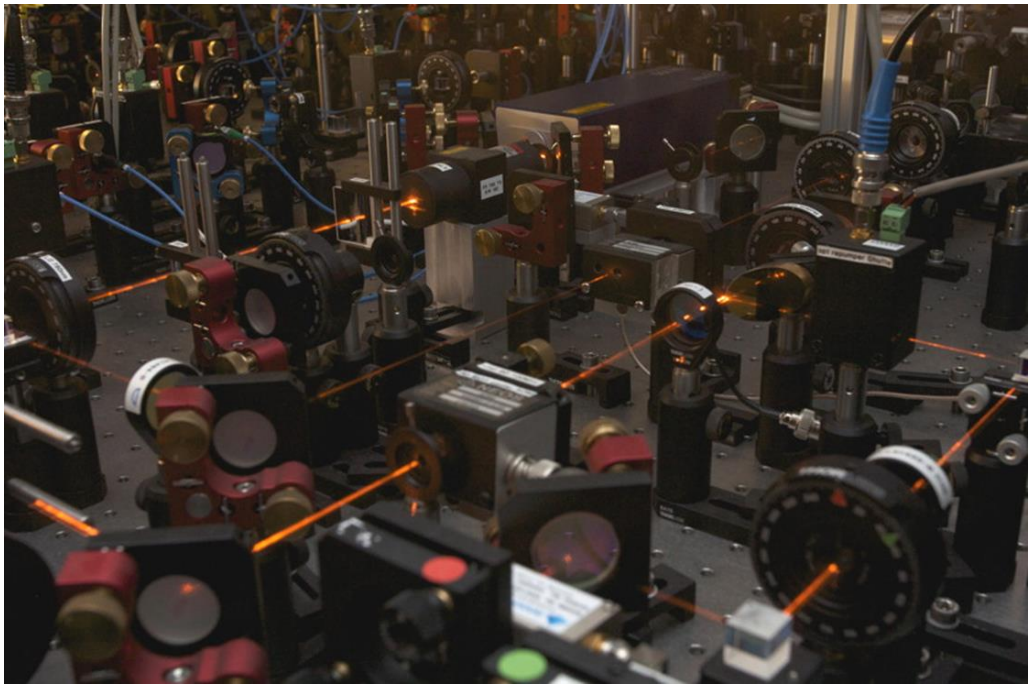
RATHER something we should strive for.

Don’t Forget!

To get this:



It really takes this:



If state is $\hat{\Pi} = |\psi\rangle\langle\psi|$,

"projections"

$$1 = |\langle\psi|\psi\rangle|^2 = \mathrm{tr}\,\hat{\Pi}\hat{I}$$

$$\overline{\sigma_x} = \langle\psi|\hat{\sigma}_x|\psi\rangle = \mathrm{tr}\,\hat{\Pi}\hat{\sigma}_x$$

$$\overline{\sigma_y} = \langle\psi|\hat{\sigma}_y|\psi\rangle = \mathrm{tr}\,\hat{\Pi}\hat{\sigma}_y$$

$$\overline{\sigma_z} = \langle\psi|\hat{\sigma}_z|\psi\rangle = \mathrm{tr}\,\hat{\Pi}\hat{\sigma}_z$$

fix the state uniquely.

$\hat{I}, \hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$ — linearly indep.

A Very Fundamental Mmt ?

Suppose d^2 projectors $\Pi_i = |\psi_i\rangle\langle\psi_i|$ satisfying

$$\mathrm{tr}\,\Pi_i \Pi_j = \frac{1}{d+1} \quad , \quad i \neq j$$

exist. ← Called SIC.

Can prove:

1) the Π_i linearly independent

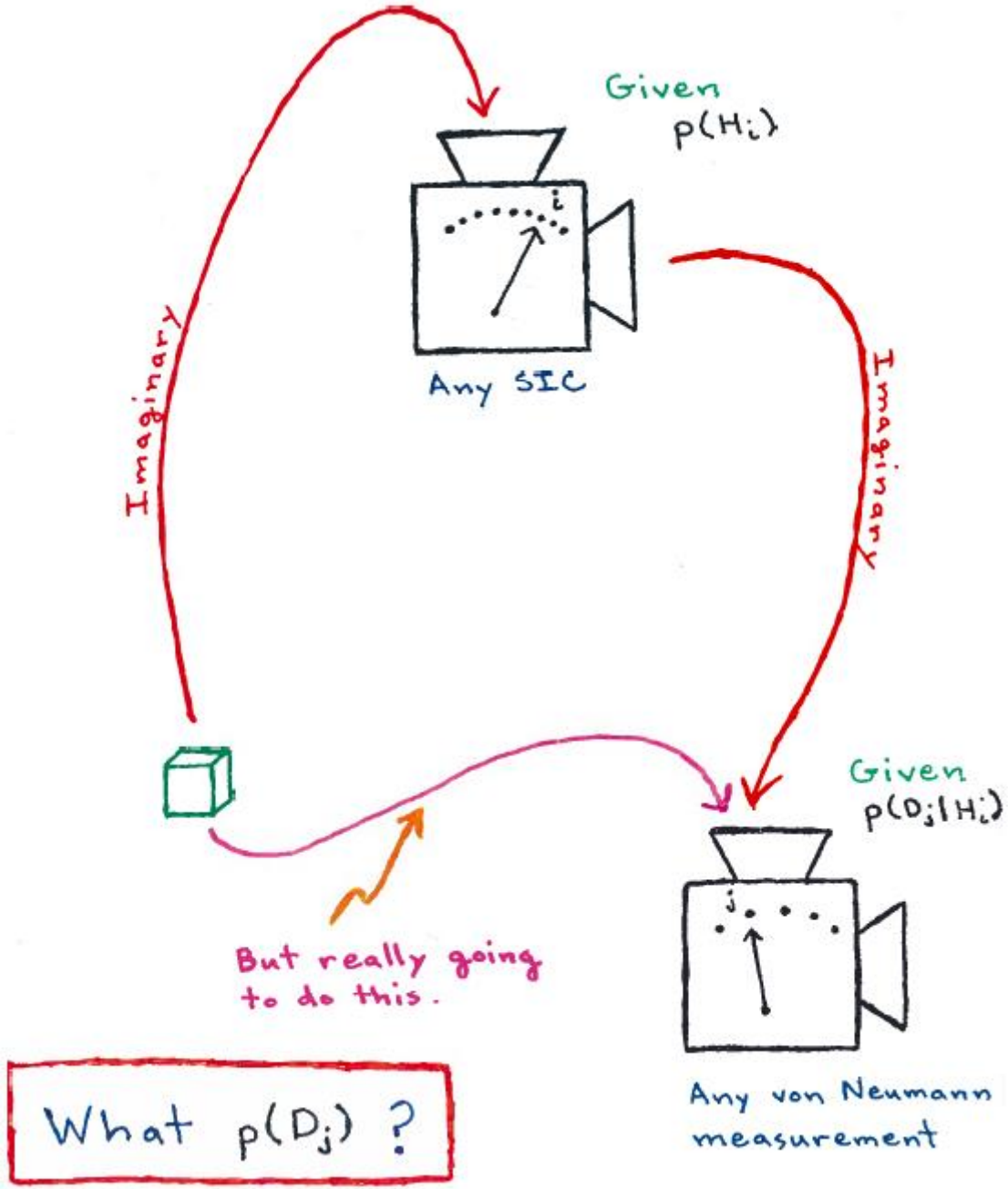
2) $\sum_i \frac{1}{d} \Pi_i = I$

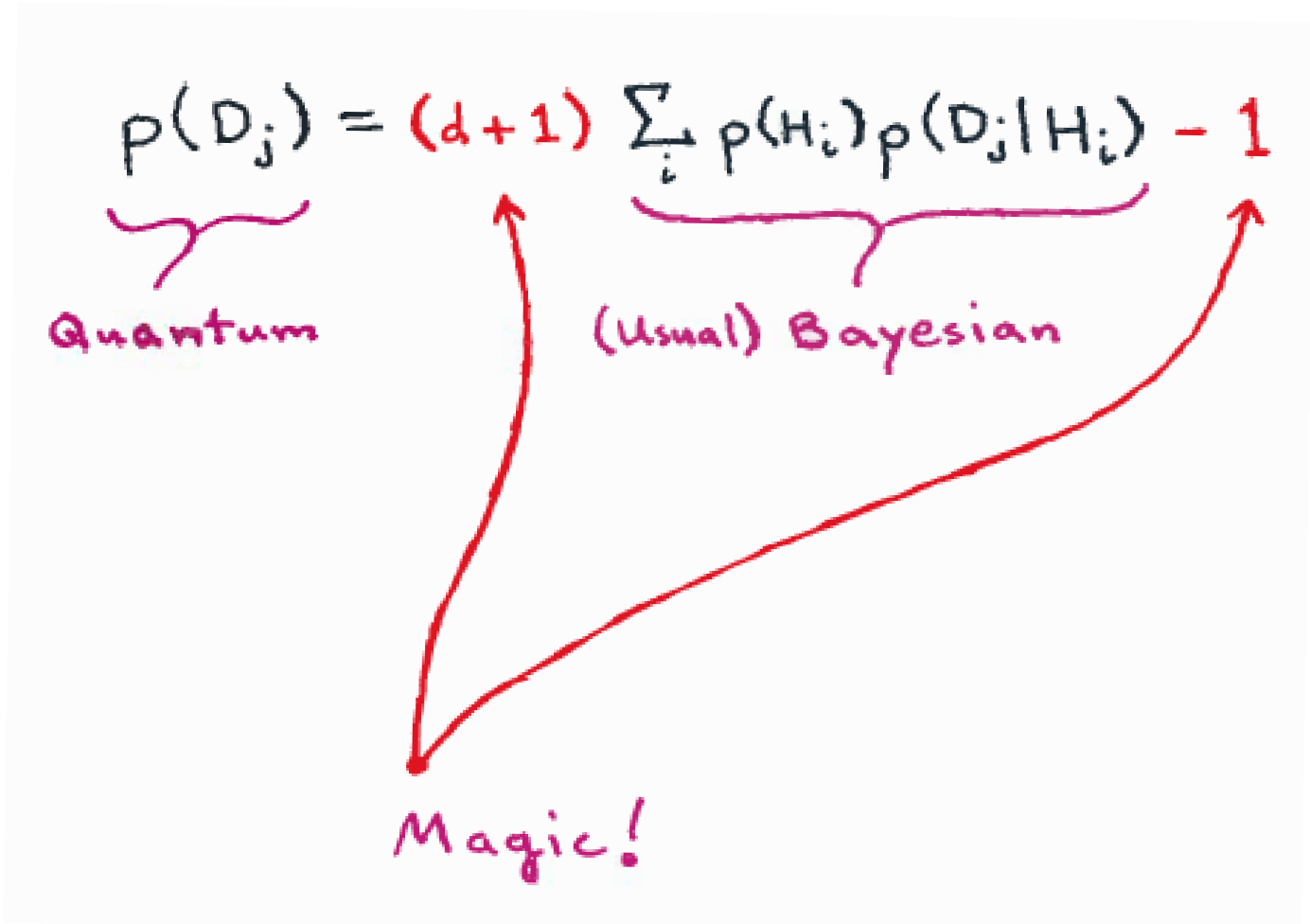
So good for Bureau of Standards.

Also

$$p(i) = \frac{1}{d} \mathrm{tr}\,\rho \Pi_i$$

$$\boxed{\rho = \sum_i \left[(d+1) p(i) - \frac{1}{d} \right] \Pi_i}$$


Given
$p(H_i)$
Any SIC
Imaginary
Imaginary
Given
$p(D_j|H_i)$
But really going to do this.
Any von Neumann measurement
What $p(D_j)$?


$p(D_j) = (d+1) \sum_i p(H_i) p(D_j|H_i) - 1$
Quantum
(Usual) Bayesian
Magic!

Generalized considerations give

$$p(D_j) = (1 + \tfrac{1}{2} q d) \sum_i p(H_i) p(D_j | H_i) - \tfrac{1}{2} q$$

$q = 0, 1, 2, \ldots$
character of the zing
(For QM, $q = 2$.)

$d = 2, 3, 4, \ldots$
value of a (local) beable, how much zing

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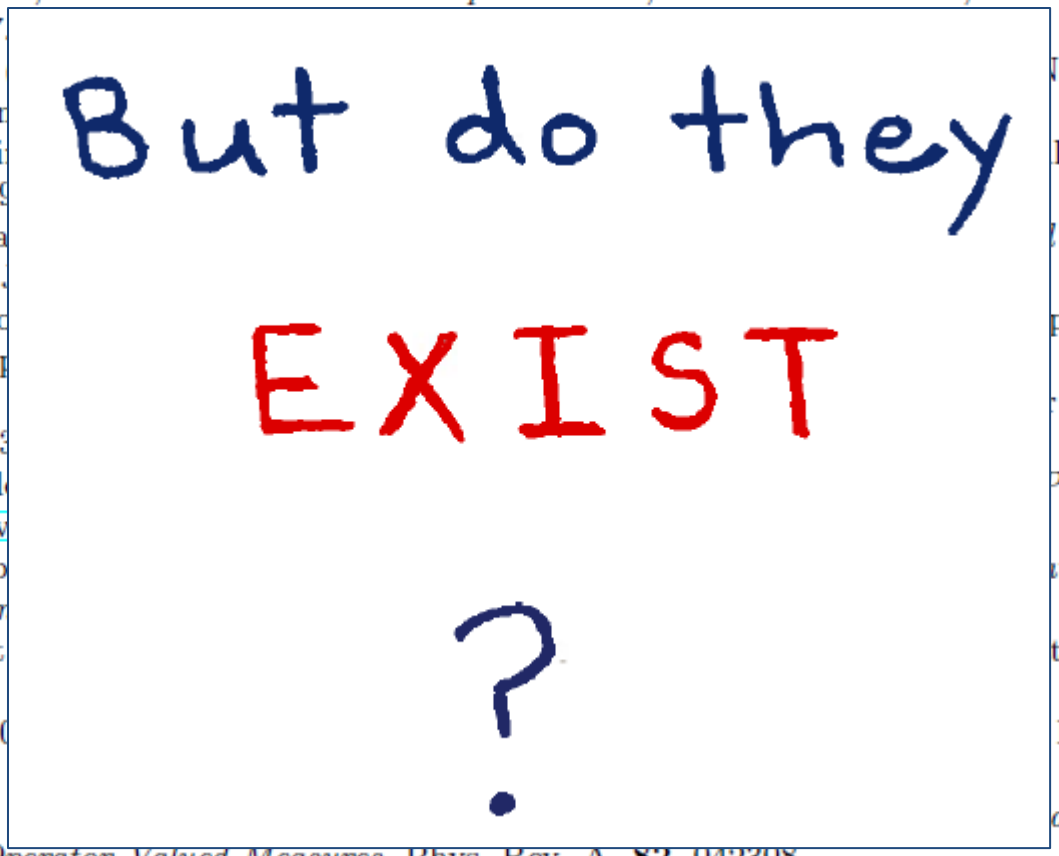

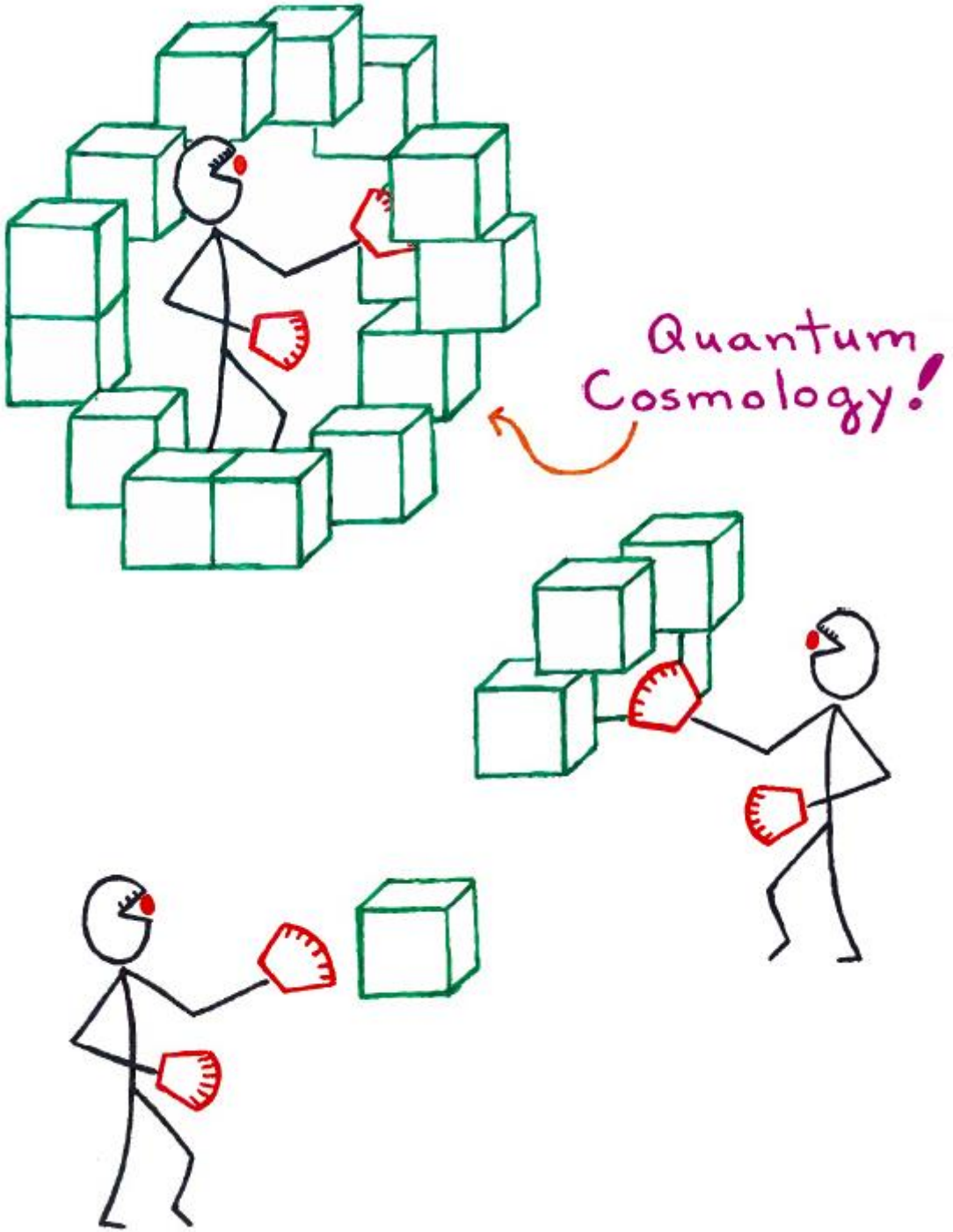
Quantum
Cosmology!

Erwin Schrödinger on Responsible Physics*

In an honest search for knowledge you quite often have to abide by ignorance for an indefinite period. Instead of filling a gap by guesswork, genuine science prefers to put up with it; and this, not so much from conscientious scruples about telling lies, as from the consideration that, however irksome the gap may be, its obliteration by a fake removes the urge to seek after a tenable answer. So efficiently may attention be diverted that the answer is missed even when, by good luck, it comes close at hand. The steadfastness in standing up to a *non liquet*, nay in appreciating it as a stimulus and a signpost to further quest, is a natural and indispensable disposition in the mind of a scientist. This in itself is apt to set him at variance with the religious aim of closing the picture, unless each of the two antagonistic attitudes, both legitimate for their respective purposes, is applied with prudence.

— Erwin Schrödinger, 1954

* This message sponsored by *QBists for Quantum Attitude Reform.*