# Parallel Lives: Why Quantum Mechanics is a Local Realistic Theory After All

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[1] All the ideas presented here originate with Paul Raymond-Robichaud.
Gilles Brassard is merely the passionate messenger!

# The Meaning and non-Meaning of Bell's Theorem

Conventional Wisdom: The violation of Bell's inequality is incompatible with local realism.

Fact: This is false!

Truth: The violation of Bell's inequality is incompatible with local hidden variable theories.
*That's Different!*

What about Quantum Mechanics? Can it be local realistic, Bell's Theorem notwithstanding?

Yes! It can! It was prophecised by Everett,
explained by Frank Tipler to Deutsch,
published by Deutsch and Hayden (2000).

Can it be done in a simple way? YES!

# Desiderata for Local Realism

- Systems should have local physical states.
- Systems should have local evolution.
- The whole should be fully described by its parts.
- Observations of a system should be determined by its physical state.

# Desiderata for Local Realism

## Warning!

By local realist we do not merely mean that no action at point A can have instantaneous *observable* effects at point B.

We mean no effect whatsoever on the state at point B.

# But WAIT!

Didn't John Bell prove in 1964 that this is impossible?!

Not really...
He proved that it's impossible by use of local hidden variables.

He also said:
*What is proved by impossibility proofs is lack of imagination*

And Einstein said:
*Imagination is more important than knowledge*

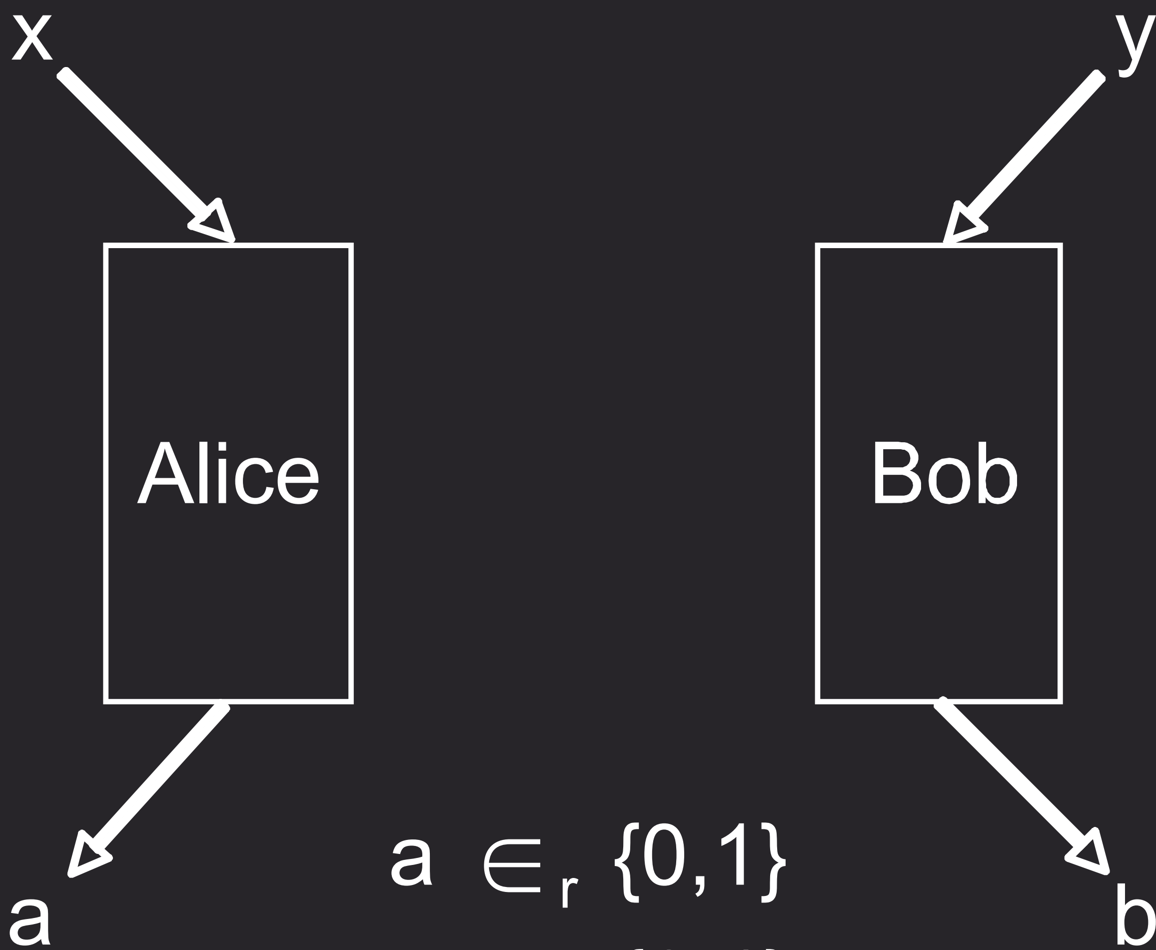
So...Could there be another local-realistic way?

One that requires just a little more imagination!

# Digression: Popescu-Rohrlich Nonlocal Boxes

- They violate a Bell inequality (CHSH) maximally.
- $\mathrm{CHSH}_{\mathrm{PR}} = 4$; $\mathrm{CHSH}_{\mathrm{QM}} = 2\sqrt{2} \approx 2.83$; $\mathrm{CHSH}_{\mathrm{Classical}} = 2$
- So, PR-boxes are even more "nonlocal" than quantum mechanics.
- Yet, they can be given a fully local-realistic explanation!
- This proves that the violation of a Bell inequality is not a proof of nonlocality… Bell's theorem notwhitstanding!

# Non-local boxes




$$a \in_r \{0,1\}$$

$$b \in_r \{0,1\}$$

$$a \oplus b = x \wedge y$$

# Non-Local Boxes

They cannot be used to communicate:
*They are causal and atemporal*

They can be simulated *classically*
with probability 75%

They can be simulated *quantumly*
with probability $\cos^2\frac{\pi}{8} = \frac{2+\sqrt{2}}{4} \approx 85\%$

# Fact about PR Boxes

- The correlations entailed by these PR Boxes provide a maximal violation of the CHSH Bell inequality.
- Hence, they cannot be explained by local hidden variables.
- Nevertheless, let's see how to "implement" them locally!
- For the sake of illustration let us take the inputs in $\{0, 1\}$ but the outputs in $\{\text{green}, \text{red}\}$

# Parallel Lives: A local realistic interpretation of "nonlocal" boxes

Gilles Brassard and Paul Raymond-Robichaud, Université de Montréal

"What is proved by impossibility proofs is lack of imagination" - John Bell
"Imagination is more important than knowledge" - Albert Einstein

## Abstract:

We show how local realism can be consistent with bipartite correlations that are usually considered to be nonlocal. For this purpose, we conduct a thought experiment in an imaginary world.

## Imaginary World:

Our imaginary world follows the principles of Locality and Realism.

Principle of Locality: No action taken at a point A can have any effect at a point B at a speed faster than light.

Principle of Realism: There is a real world and observations are determined by the state of the real world.

This world has two inhabitants, Alice and Bob, which are each carrying a PR box, introduced by Popescu and Rorhlich.


A PR box has a "0" and a "1" button. Whenever a button is pushed, it instantaneously flashes a red or green light with equal probability. If Alice and Bob both push a button, they will discover when they meet that they have seen different colours precisely when they both have pushed the "1" button.

*(Note that the PR box does not enable instantaneous communication between Alice and Bob)*

## Alice and Bob will test the boxes with this protocol:

They travel far apart in their spaceships. Alice and Bob flip coins and push the corresponding button simultaneously.



Once a button is pushed, the box flashes either a green or red light.

The experiment is performed with sufficient simultaneity that Alice's box cannot know the result of Bob's coin flip (hence the input to Bob's box) before it has to flash its own light, and vice versa.

After many experiments, they meet and realize that the boxes work perfectly.

## The Einstein-Podolsky-Rosen Argument:

- Alice's pushing of a button cannot have any instantaneous effect on Bob's system by the principle of Locality.
- After Alice pushes her button, she can know with certainty what colour Bob will see depending on which button he pushes. *(For example, if Alice pushes "1" and sees green, she knows that if Bob pushes "0" he will see green)*
- Since it is possible for Alice to predict with certainty what light Bob will see when he pushes a button, without influencing his system, it means that his observations were predetermined.
- The observations of Bob should be described by local hidden variables B0 and B1.

  B0 = 0 if Bob will observe green after pushing "0"
  B0 = 1 if Bob will observe red after pushing "0"
  B1 = 0 if Bob will observe green after pushing "1"
  B1 = 1 if Bob will observe red after pushing "1"

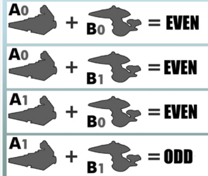
- Likewise, Alice's system should be described by local hidden variables A0 and A1.

  A0 = 0 if Alice will observe green after pushing "0"
  A0 = 1 if Alice will observe red after pushing "0"
  A1 = 0 if Alice will observe green after pushing "1"
  A1 = 1 if Alice will observe red after pushing "1"

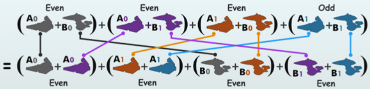
- A local hidden variable theory would give a local realistic explanation of this experiment.

## Bell's Theorem: Local hidden variable theories can only produce PR boxes that work at most 75% of the time.

Proof: A hidden variable theory of these boxes must satisfy the following 4 equations:




Summing these equations on both sides and rearranging the terms:




This implies: Even = Odd!

It is not possible for the four equations to be all correct. At least one of the four possible choices of buttons pushed will give incorrect results.

Many people have concluded that any world that could produce PR boxes that work more than 75% of the time cannot be Local and Realistic. Remarkably, quantum mechanics enables PR boxes that work 85% of the time. Must we conclude that quantum mechanics cannot be Local and Realistic?

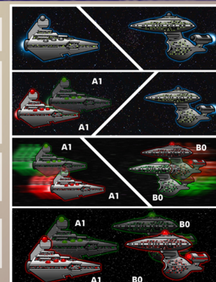
## Here is how the seemingly impossible is accomplished:

Each spaceship lives inside a bubble.

When Alice pushes a button on her box (here "1"), her bubble splits into two bubbles. Each bubble contains a copy of its spaceship and its inhabitant. Inside one bubble, Alice has seen the red light flash; inside the other, she has seen the green light flash. From now on, the two bubbles are living **parallel lives**. They cannot interact in any way and will never meet again. Notice that this phenomenon is strictly local.

The same phenomenon takes place when Bob pushes his button (here "0") on the box. Let's see what happens when they travel toward each other.

Each of the two bubbles that contain Alice is allowed to interact with and see only a single bubble that contains Bob, namely the one that satisfies the equations described above.




Note that such a perfect matching is always possible. Furthermore, each bubble can "know" with which other bubble to interact provided it keeps a local memory of which button was pushed and which light flashed. Alice and Bob will be under the illusion of correlations that seem to emerge from outside of space and time.

In our imaginary world, the Einstein-Podolsky-Rosen argument does not hold because whenever Alice pushes a button and can predict something about Bob, she is really predicting, not what is happening simultaneously at Bob's place, but how their various lives will meet in the future.

## Conclusion:

The virtue of our imaginary world is to demonstrate in an exceedingly simple way that local reality can produce correlations that are impossible in any classical theory based on local hidden variables.

In quantum mechanics, a theory analogous to this one can be traced back to at least Deutsch and Hayden.

**Perhaps we live parallel lives?**

## References:

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Brassard, G. and Raymond-Robichaud, P. (2013). Can free will emerge from determinism in quantum theory? Chapter 4 in *Is Science Compatible with Free Will? Exploring Free Will and Consciousness in the Light of Quantum Physics and Neuroscience*. Suarez, A. and Adams, P. (eds), Springer. Also available at: http://arxiv.org/abs/1204.2128.

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Einstein, A., Podolsky, B. and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **47**:777-780.

Popescu, S. and Rorhlich, D. (1994) Quantum nonlocality as an axiom. *Foundations of Physics* **24**(3):379-385.

Poster art and design by Louis Fernet-Leclair (2012)

## Imaginary World:

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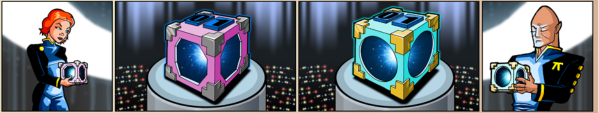
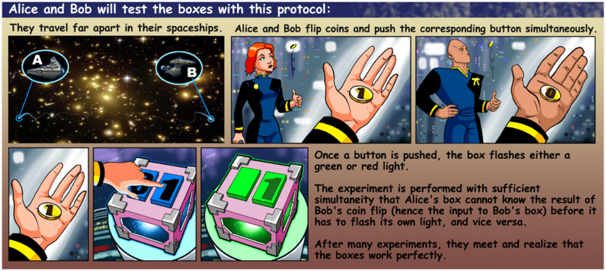
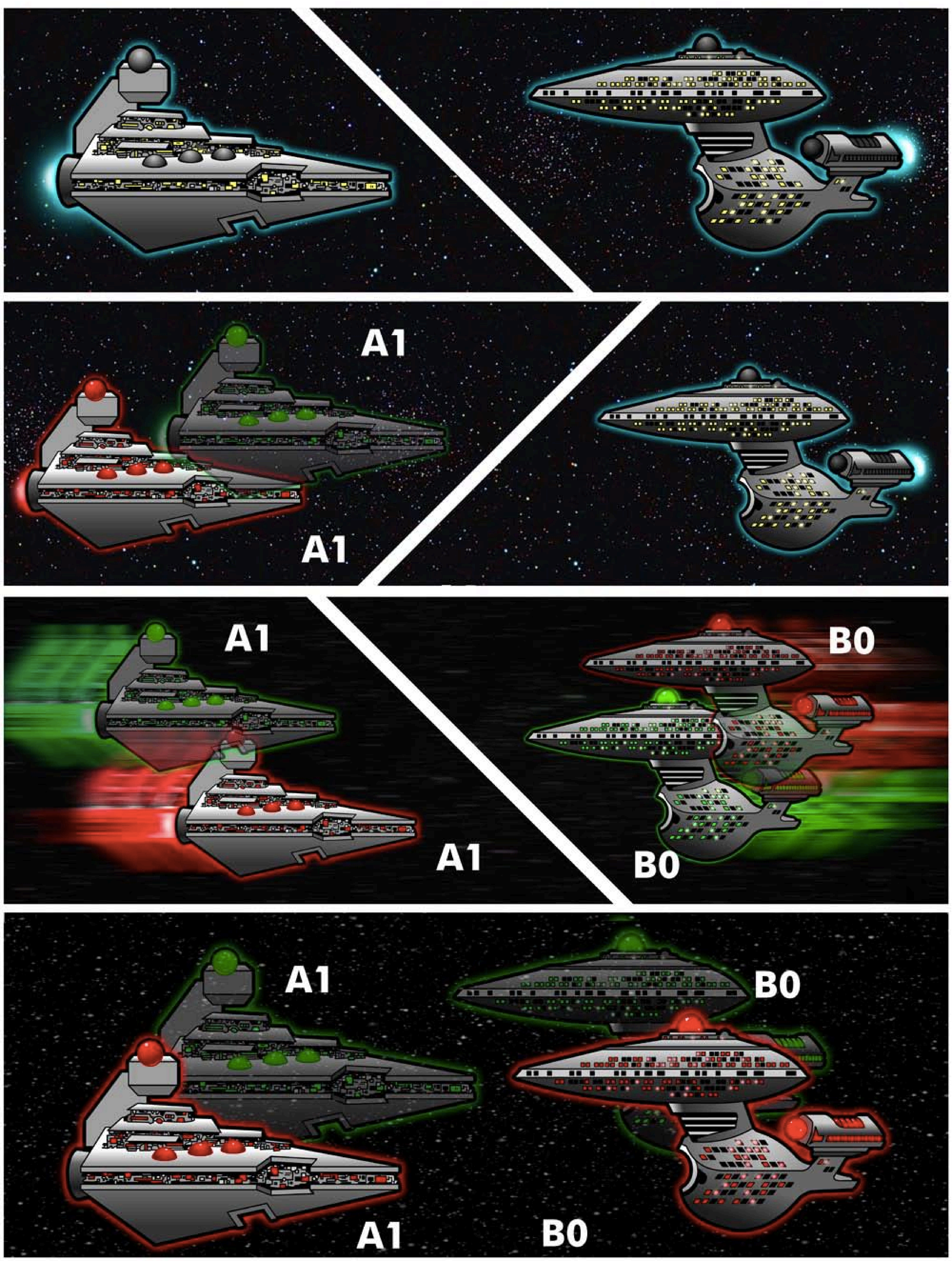
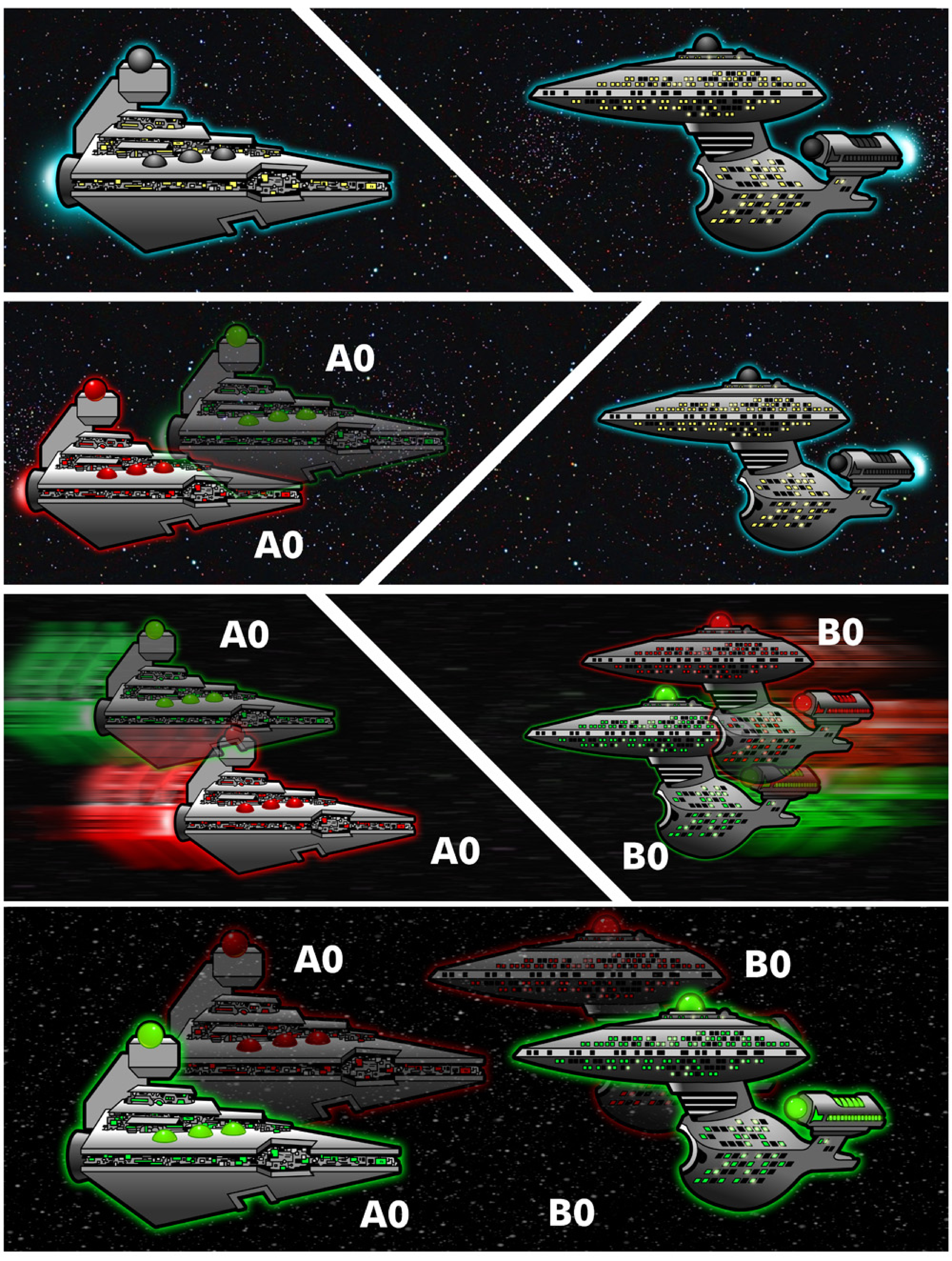


A PR box has a "0" and a "1" button. Whenever a button is pushed, it instantaneously flashes a red or green light with equal probability. If Alice and Bob both push a button, they will discover when they meet that they have seen different colours precisely when they both have pushed the "1" button.

*(Note that the PR box does not enable instantaneous communication between Alice and Bob)*


Alice and Bob will test the boxes with this protocol:
They travel far apart in their spaceships.
A
B
Alice and Bob flip coins and push the corresponding button simultaneously.
1
0
1
0
1
1
Once a button is pushed, the box flashes either a green or red light.
The experiment is performed with sufficient simultaneity that Alice's box cannot know the result of Bob's coin flip (hence the input to Bob's box) before it has to flash its own light, and vice versa.
After many experiments, they meet and realize that the boxes work perfectly.


A1
A1
A1
B0
A1
B0
A1
B0
A1
B0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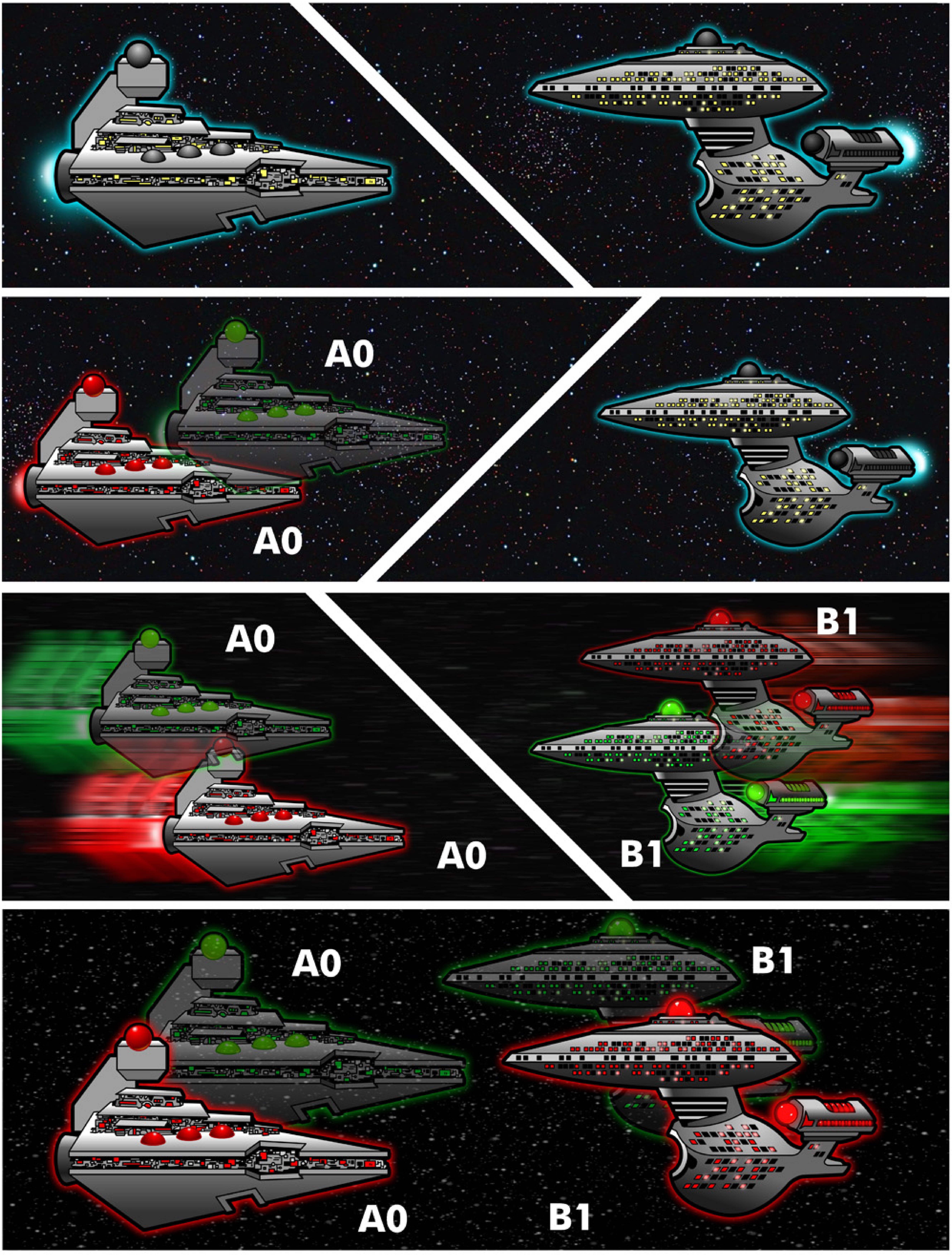
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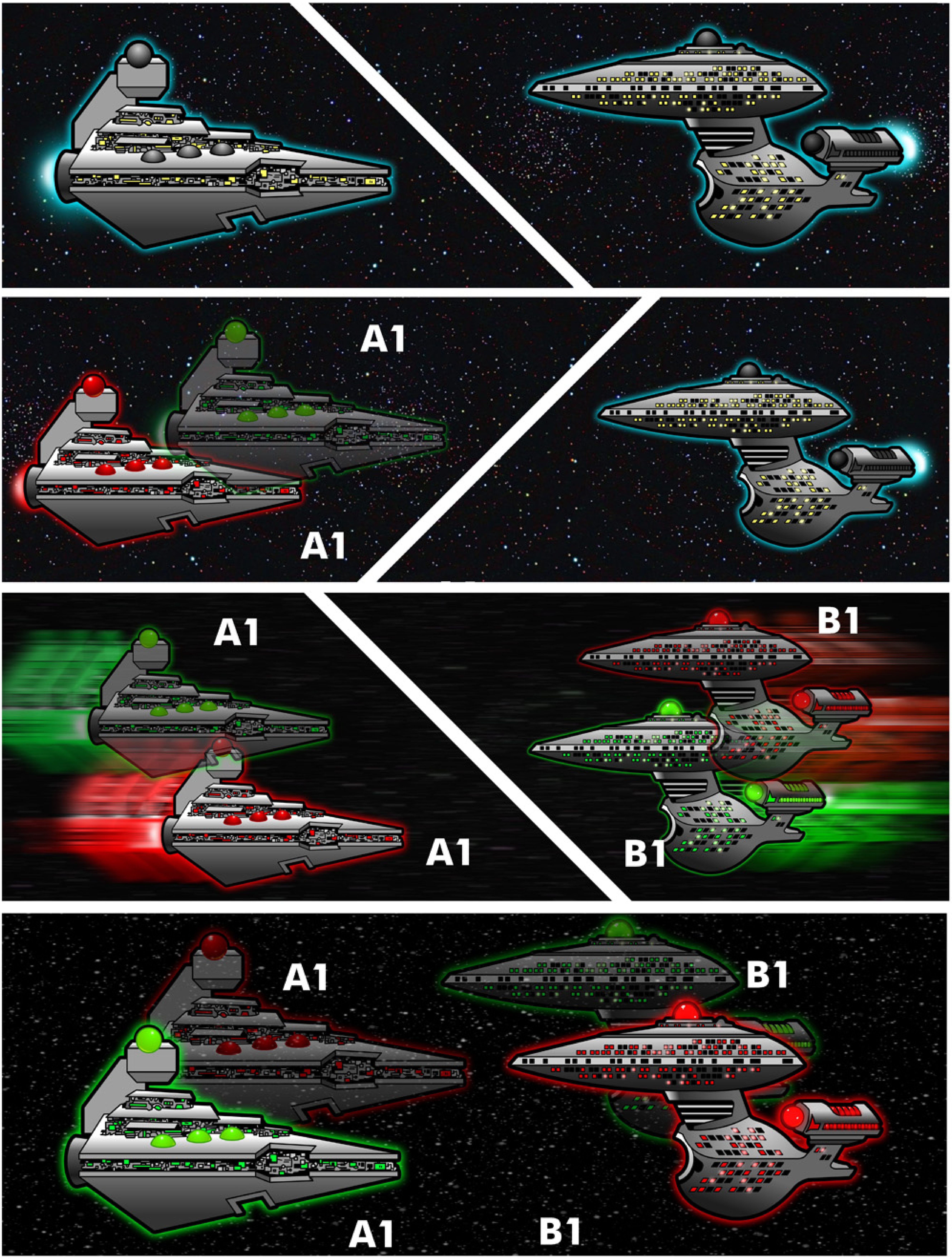
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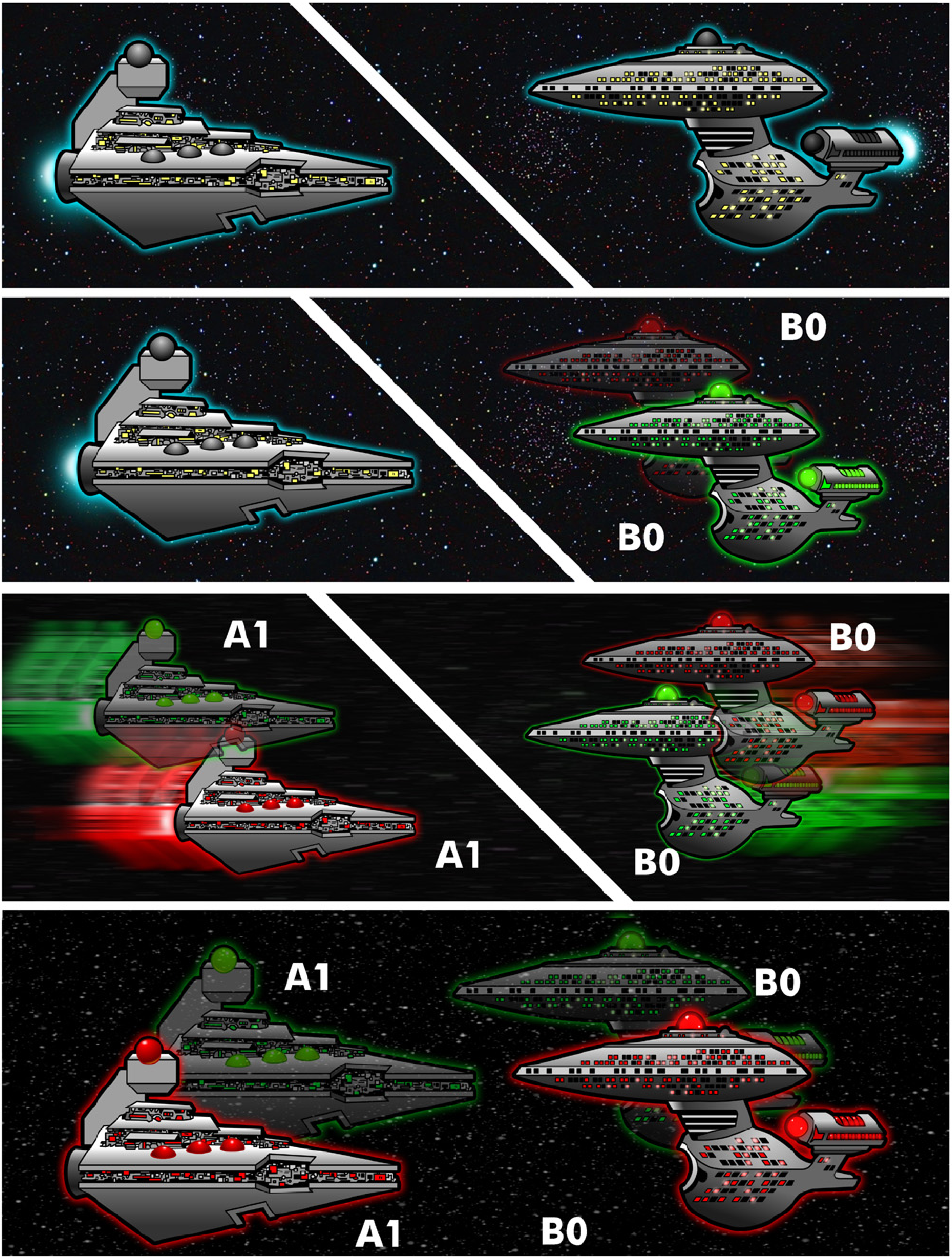
B1


B0
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A1
B0

# The key idea

In our imaginary world, the Einstein-Popolsky-Rosen argument does not hold because whenever Alice pushes a button and can predict something about Bob, she is really predicting not what is happening simultaneously at Bob's placebut how their various lives will meet in the future.

http://www.iro.umontreal.ca/~brassard/Bell/poster.jpg

This proves that

it is wrong to claim that

any world that violates

Bell inequalities

has to be nonlocal

http://www.iro.umontreal.ca/~brassard/Bell/poster.jpg

# Desiderata for Local Realism

- Systems should have local physical states.
- Systems should have local evolution.
- The whole should be fully described by its parts.
- Observations of a system should be determined by its physical state.

# Desiderata for Local Realistic Quantum Mechanics

- Systems should have local physical states.
- Systems should have local evolution.
- The whole should be fully described by its parts.
- Observations of a system should be determined by its physical state *and* be the same as those of quantum mechanics.
- PR-nonlocal boxes do *not* fulfil this last condition.
- But *we* can! :-)

# Local States

## Complete Description

For any system $X$, let $M^X$ denote its complete description.

# Local States

## Separation

The parts come from the whole:

$$M^A = \mathrm{tr}_B \left( M^{AB} \right)$$

# Local States

## Merging

If we have a system $A$ and a system $B$, it is possible to join them and form a composite system $AB$.

The state of the composite system is completely determined by the state of its parts:

$$M^{AB} = M^{A} \odot M^{B} \, .$$

**Even for entangled states!**

P.S. This is the point of this talk!

# Local Evolution

## Evolution of Isolated Systems

If we apply an operation $U$ to an isolated system $A$ that was in state $M_1^A$ , the new state of the system $M_2^A$ will be determined only by its previous state and the operation.

$$M_2^A = U\left(M_1^A\right)$$

# Local Evolution

## Evolution is a Group Action

$$
\begin{aligned}
V\left(U\left(M^A\right)\right) &= (VU)\left(M^A\right) \\
I\left(M^A\right) &= M^A
\end{aligned}
$$

# Local Evolution

## Separate Evolution

If we apply $U$ to system $A$ and $V$ to system $B$, the resulting joint state can be obtained by purely local operations on $A$ and $B$:

$$(U \otimes V)\left(M^{AB}\right) = U\left(M^{A}\right) \odot V\left(M^{B}\right)$$

# Observations in Quantum Mechanics

The axioms must give rise to the same *observations* as those in standard quantum mechanics.

However, these observations are mere individual *perceptions*, which are explained to be unavoidable by the theory itself.

The density matrix encompasses all that is observable about a system.

Observations are determined by the state of the system.

It follows that the density matrix must be a function of the physical state of the system:

$$\rho^A = f\left(M^A\right) .$$

# Observations in Quantum Mechanics

## The Density Matrix

$$U\left(f\left(M^A\right)\right) = f\left(U\left(M^A\right)\right)$$

$$\begin{tikzcd}
M^A \arrow[r, "U"] \arrow[d, "f"'] & U\left(M^A\right) \arrow[d, "f"] \\
\rho^A \arrow[r, "U"'] & U\left(\rho^A\right)
\end{tikzcd}$$

Here, $U\left(\rho^A\right) = U\rho^A U^\dagger$, but what of $U\left(M^A\right)$?
This will be defined soon!

# Observations in Quantum Mechanics

## The Density Matrix

$$f\left(\mathrm{tr}_B\left(M^{AB}\right)\right) = \mathrm{tr}_B\left(f\left(M^{AB}\right)\right)$$

$$\begin{tikzcd}
M^{AB} \arrow[r, "\mathrm{tr}_B"] \arrow[d, "f"'] & M^A \arrow[d, "f"] \\
\rho^{AB} \arrow[r, "\mathrm{tr}_B"'] & \rho^A
\end{tikzcd}$$

Again, $\mathrm{tr}_B(\rho^{AB})$ is the usual partial trace in quantum mechanics, whereas $\mathrm{tr}_B(M^{AB})$ will be defined soon!

# Parallel Lives vs Many Worlds

The universal wave function cannot be the complete description of a local universe. It merely describes what can be observed.

The universal wave function is but a shadow of the real world!

# Parallel Lives vs Many Worlds

## Bell States and Bit flips: A reminder

Two of the four Bell States:

$$\left|\Psi^+\right\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}\left(|11\rangle + |00\rangle\right)$$

The negation gate:

$$N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

# Parallel Lives vs Many Worlds

According to the Desiderata, all states are separable:

$$\left|\Psi^+\right\rangle = M^A \odot M^B$$

By separate evolution:

$$(N \otimes N)\left|\Psi^+\right\rangle = N\left(M^A\right) \odot N\left(M^B\right)$$

Since

$$\left|\Psi^+\right\rangle = (N \otimes N)\left|\Psi^+\right\rangle$$

By tracing out $B$, we conclude

$$M^A = N\left(M^A\right)$$

However

$$\left|\Phi^+\right\rangle = (N \otimes I)\left|\Psi^+\right\rangle = N\left(M^A\right) \odot M^B = M^A \odot M^B = \left|\Psi^+\right\rangle$$

A contradiction!

# States

For a system $A$ associated with a Hilbert Space of dimension $n$, its state $M^A$ is formally defined by an *evolution matrix* $[W]^A$, which is an $n \times n$ matrix whose entries are matrices:

$$[W]^A_{i,j} \stackrel{\text{def}}{=} W^\dagger \left( |j\rangle\langle i| \otimes I^{\overline{A}} \right) W$$

for some unitary $W$ on the global state, which corresponds to all that happened to the universe since the beginning of time.

# Local Evolution

If we have a system $A$ in state $\left[W\right]^A$ on which we apply a unitary operation $U$, the system evolves to

$$U\left[W\right]^A$$

defined as

$$\left(U\left[W\right]^A\right)_{i,j} \stackrel{\text{def}}{=} \sum_{m,n} U_{i,m}\left[W\right]^A_{m,n} U^\dagger_{n,j}$$

**Theorem**

$$U\left[W\right]^A = \left[\left(U \otimes V\right) W\right]^A$$

for any unitary $V$ acting on $\overline{A}$

# Local Evolution

**Proof**

$$
\begin{aligned}
\left(U\left[W\right]^{A}\right)_{i,j} &= \sum_{m,n} U_{i,m}\left[W\right]^{A}_{m,n} U^{\dagger}_{n,j} \\
&= \sum_{m,n} \langle i|U|m\rangle \left(W^{\dagger}\left(|n\rangle\langle m| \otimes I^{\bar{A}}\right) W\right) \langle n|U^{\dagger}|j\rangle \\
&= \sum_{m,n} W^{\dagger}\left(\left(|n\rangle \langle n|U^{\dagger}|j\rangle \langle i|U|m\rangle \langle m|\right) \otimes I^{\bar{A}}\right) W \\
&= W^{\dagger}\left(\left(\sum_{m,n} |n\rangle\langle n| \; U^{\dagger}|j\rangle\langle i|U \; |m\rangle\langle m|\right) \otimes I^{\bar{A}}\right) W \\
&= W^{\dagger}\left(\left(U^{\dagger} \, |j\rangle\langle i| \, U\right) \otimes I^{\bar{A}}\right) W \\
&= W^{\dagger}\left(\left(U^{\dagger} \, |j\rangle\langle i| \, U\right) \otimes \left(V^{\dagger} I^{\bar{A}} V\right)\right) W \\
&= W^{\dagger}\left(U^{\dagger} \otimes V^{\dagger}\right)\left(|j\rangle\langle i| \otimes I^{\bar{A}}\right)\left(U \otimes V\right) W \\
&= \left[\left(U \otimes V\right) W\right]^{A}_{i,j}
\end{aligned}
$$

# Separation

The evolution matrix of a system $A$ can be obtained from the evolution matrix of a system $AB$ by a trace operation defined as

$$\left(tr_B\left[W\right]^{AB}\right)_{i,j} \stackrel{\text{def}}{=} \sum_k \left[W\right]^{AB}_{(i,k),(j,k)}$$

**Theorem**

$$\left[W\right]^{A} = \mathrm{tr}_B\left[W\right]^{AB}.$$

# Separation

**Proof**

$$\begin{aligned}
\left(\mathrm{tr}_B\left[W\right]^{AB}\right)_{i,j} &= \sum_k \left[W\right]^{AB}_{(i,k),(j,k)} \\
&= \sum_k W^\dagger \left(|j\rangle\langle i|^A \otimes |k\rangle\langle k|^B \otimes I^{\overline{AB}}\right) W \\
&= W^\dagger \left(|j\rangle\langle i|^A \otimes I^B \otimes I^{\overline{AB}}\right) W \\
&= \left[W\right]^A_{i,j}
\end{aligned}$$

# Merging

The state of a joint system $AB$ can be obtained from the evolution matrices of systems $A$ and $B$ by the joint product $\odot$ defined as

$$\left( \left[ W \right]^{A} \odot \left[ W \right]^{B} \right)_{(i,k),(j,l)} \stackrel{\text{def}}{=} \left[ W \right]^{A}_{i,j} \left[ W \right]^{B}_{k,l}$$

**Theorem**

$$\left[ W \right]^{AB} = \left[ W \right]^{A} \odot \left[ W \right]^{B}.$$

# Merging

**Proof**

$$\left(\left[W\right]^{A}\odot\left[W\right]^{B}\right)_{(i,k),(j,l)}$$

$$=\left[W\right]^{A}_{i,j}\left[W\right]^{B}_{k,l}$$

$$=\left(W^{\dagger}\left(|j\rangle\langle i|^{A}\otimes I^{B}\otimes I^{\overline{AB}}\right)W\right)\left(W^{\dagger}\left(I^{A}\otimes|l\rangle\langle k|^{B}\otimes I^{\overline{AB}}\right)W\right)$$

$$=W^{\dagger}\left(|j\rangle\langle i|^{A}\otimes I^{B}\otimes I^{\overline{AB}}\right)\left(I^{A}\otimes|l\rangle\langle k|^{B}\otimes I^{\overline{AB}}\right)W$$

$$=W^{\dagger}\left(|j\rangle\langle i|^{A}\otimes|l\rangle\langle k|^{B}\otimes I^{\overline{AB}}\right)W$$

$$=\left[W\right]^{AB}_{(i,k),(j,l)}$$

# Recovering the Density Matrix

Let $|\psi\rangle$ be a vector in the Hilbert space of the universe.
We define $\left[W\right]^{A}|\psi\rangle$ by:

$$\left(\left[W\right]^{A}|\psi\rangle\right)_{i,j} \stackrel{\text{def}}{=} \langle\psi|\left[W\right]^{A}_{i,j}|\psi\rangle$$

**Theorem**

$$\left[W\right]^{A}|\psi\rangle = \left[W|\psi\rangle\right]^{A}$$

where

$$\left[W|\psi\rangle\right]^{A} \stackrel{\text{def}}{=} tr_{\overline{A}}\left(W|\psi\rangle\langle\psi|W^{\dagger}\right).$$

We call $|\psi\rangle$ the *reference vector*.

# Recovering the Density Matrix

**Proof**

$$
\begin{aligned}
\left( [W]^{A} |\psi\rangle \right)_{i,j} &= \langle\psi| \, [W]^{A}_{i,j} \, |\psi\rangle \\
&= \langle\psi| \left( W^{\dagger} \left( |j\rangle\langle i| \otimes I^{\overline{A}} \right) W \right) |\psi\rangle \\
&= \langle\psi| \left( W^{\dagger} \left( |j\rangle\langle i| \otimes \sum_{k} |k\rangle\langle k| \right) W \right) |\psi\rangle \\
&= \sum_{k} \left( \langle\psi| \, W^{\dagger} \, |j\rangle|k\rangle \right) \left( \langle i|\langle k| \, W \, |\psi\rangle \right) \\
&= \sum_{k} \langle i|\langle k| \left( W \, |\psi\rangle\langle\psi| \, W^{\dagger} \right) |j\rangle|k\rangle \\
&= \left( tr_{\overline{A}} \left( W \, |\psi\rangle\langle\psi| \, W^{\dagger} \right) \right)_{i,j} = [W \, |\psi\rangle]^{A}_{i,j}
\end{aligned}
$$

# Proportion versus Probability

The *proportion* of a system $A$, with evolution Matrix $[W]^A$ in state $|i\rangle$ is given by

$$\Big[ W \, |\psi\rangle \Big]_{i,i}$$

where $|\psi\rangle$ is the reference vector.

This proportion is *perceived* as a probability, giving rise to Born's rule!

# Separate evolution

**Theorem**

$$\Big((U \otimes V)\,[W]^{AB}\Big) = U[W]^{A} \odot V[W]^{B}$$

**Proof**

$$\begin{aligned}
&\Big((U \otimes V)\,[W]^{AB}\Big) \\
=&\big[(U \otimes V \otimes I)\,W\big]^{AB} \\
=&\big[(U \otimes V \otimes I)\,W\big]^{A} \odot \big[(U \otimes V \otimes I)\,W\big]^{B} \\
=&U\,[W]^{A} \odot V\,[W]^{B}
\end{aligned}$$

# Commuting Diagrams

And indeed we have

$$\begin{tikzcd} M^A \arrow[r, "U"] \arrow[d, "f"'] & U\left(M^A\right) \arrow[d, "f"] \\ \rho^A \arrow[r, "U"'] & U\left(\rho^A\right) \end{tikzcd}$$

Which means:

$$U\big(\left[W\right]^A |\psi\rangle\big) = \left(U\left[W\right]^A\right)|\psi\rangle$$

# Commuting Diagrams

**Theorem**

$$U([W]^A\,|\psi\rangle) = (U[W]^A)\,|\psi\rangle$$

**Proof**

$$\begin{aligned}
& U\left([W]^A\,|\psi\rangle\right) \\
=& U[W\,|\psi\rangle]^A \\
=& [(U\otimes I)W\,|\psi\rangle]^A \\
=& [(U\otimes I)W]^A\,|\psi\rangle \\
=& (U[W]^A)\,|\psi\rangle
\end{aligned}$$

# Commuting Diagrams

As well as

$$\begin{tikzcd} M^{AB} \arrow[r, "\mathrm{tr}_B"] \arrow[d, "f"'] & M^{A} \arrow[d, "f"] \\ \rho^{AB} \arrow[r, "\mathrm{tr}_B"'] & \rho^{A} \end{tikzcd}$$

Which means:

$$\mathrm{tr}_B\Big( \big[W\big]^{AB} |\psi\rangle \Big) = \Big( \mathrm{tr}_B \big[W\big]^{AB} \Big) |\psi\rangle$$

# Commuting Diagrams

**Theorem**

$$\mathrm{tr}_B\Big( \big[\, W \,\big]^{AB} \, |\psi\rangle \Big) = \Big( \mathrm{tr}_B \big[\, W \,\big]^{AB} \Big) \, |\psi\rangle$$

**Proof**

$$\begin{aligned} & \mathrm{tr}_B\Big( \big[\, W \,\big]^{AB} \, |\psi\rangle \Big) \\ =& \mathrm{tr}_B\Big( \big[\, W \, |\psi\rangle \,\big]^{AB} \Big) \\ =& \big[\, W \, |\psi\rangle \,\big]^{A} \\ =& \big[\, W \,\big]^{A} \, |\psi\rangle \\ =& \Big( \mathrm{tr}_B \big[\, W \,\big]^{AB} \Big) \, |\psi\rangle \end{aligned}$$

# Conclusion

Our work improves on Everettian axioms in the following ways:

- The universal wave function cannot be the complete description of reality AND be local.
- Our system of axioms is simple, local, realistic, deterministic and complete.

# References


- D. Deutsch and P. Hayden, "Information flow in entangled quantum systems", *Proceedings of the Royal Society of London* **A456**(1999):1759–1774, 2000.
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"About your cat, Mr. Schrödinger—I have good news and bad news."