IOP PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **25** (2008) 154002 (12pp) doi:10.1088/0264-9381/25/15/154002

A general theoretical framework for decoherence in open and closed systems

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Received 14 January 2008, in final form 28 March 2008
Published 21 July 2008
Online at stacks.iop.org/CQG/25/154002

Abstract

A general theoretical framework for decoherence is proposed, which encompasses formalisms originally devised to deal just with open or closed systems. The conditions for decoherence are clearly stated and the relaxation and decoherence times are compared. Finally, the spin-bath model is developed in detail from the new perspective.

PACS number: 03.65.Yz

1. Introduction

The specific feature of quantum mechanics is the superposition principle, which leads to the phenomenon of interference. Therefore, any attempt to account for the emergence of classicality from quantum behavior must explain how interference vanishes. The process that cancels interference and selects the candidates for classical states is usually called *decoherence*.

From a diachronic perspective, the decoherence program finds its roots (though, of course, not under this name) in the attempts to explain how a coherent pure state becomes a final decohered mixture with no interference terms. Three general periods can be identified in the development of this program:

- *Closed systems period* [1–4]. In order to understand how classical macroscopic features arise from quantum microscopic behavior, 'gross' observables are defined. The states, indistinguishable for a macroscopic observer, are described by the same coarse-grained state $\rho_G(t)$. When the evolution of $\rho_G(t)$ (or of the expectation value of the gross observables) is studied, it can be proved that $\rho_G(t)$ approaches a final stable state through

a process with characteristic time t_C; therefore, $\rho_G(t)$ decoheres in its own eigenbasis after a decoherence time $t_D = t_C$. The main problem of this period was the fact that t_C turned out to be too long to account for experimental data (see [5]).

- *Open systems period*. An open system S is considered in interaction with its environment E, and the evolution of the reduced state $\rho_S(t) = \mathrm{Tr}_E \rho_{SE}(t)$ is studied. The so-called *environment induced decoherence* (EID) approach [6–11] proves that, since the interference terms of $\rho_S(t)$ rapidly vanish, $\rho_S(t)$ decoheres in an adequate pointer basis after an extremely short decoherence time $t_D = t_{DS}$. This result overcomes the main problem of the first period.

  However, the foundations of the EID program are still threatened by certain conceptual problems derived from its open-system perspective:

  - *The closed-system problem*: If only open systems may decohere, the issue of the emergence of classicality in closed systems, in particular, in the universe as a whole, cannot even be posed (see Zurek in [12]; for criticisms, see [13]).
  - *The defining-system problem*: Since the environment may be external or internal, there is no general criterion to decide where to place the 'cut' between system and environment (see Zurek's formulation of this problem in [14], and a discussion in [15]).
- *Closed and open systems period.* Although at present EID is still considered the 'orthodox' view about the matter [16, 17], in recent times other approaches have been proposed to face the conceptual difficulties of EID, in particular, the closed-system problem [18–24]. Some of these methods are clearly 'non-dissipative' [25–28]. Among them, we have developed the *self-induced decoherence* (SID) approach, according to which a closed quantum system with continuous spectrum may decohere by destructive interference, and may reach a decohered state where the classical limit can be obtained [15, 29–39].

Although the conceptual problems of EID may not be obstacles to local applications of the theory, they turn out to be serious challenges in cosmology, where the purpose is to explain the classical behavior of the universe, which, by definition, has no external environment. The usual strategy in cosmology consists of splitting the universe into some degrees of freedom which represent the 'system' of interest, and the remaining degrees of freedom that are supposed to be non-accessible and, therefore, play the role of an internal environment. For instance, in quantum field theory, when it is known that the background field follows a simple classical behavior, the scalar field is decomposed according to $\phi = \phi_c + \phi_q$, where the background field ϕ_c plays the role of the system and the fluctuation field ϕ_q plays the role of the environment (see [40]). This means that the observables which will behave classically must be assumed in advance: there is no general criterion to discriminate between system and environment. This explains the search for an account of decoherence in closed systems applicable to cosmology. For instance, in [31] we attempted to explain the emergence of classicality in a Robertson–Walker universe from the perspective of the SID approach.

In spite of the fact that, at present, formalisms for closed and open systems coexist, in the literature both kinds of approaches are often presented as alternative scenarios for decoherence, or even as theories dealing with different physical phenomena [41]. In the following sections, we will challenge this common view by showing that both approaches can be understood in the context of a general theoretical framework.

2. A general framework for decoherence

As emphasized by Omnès [42, 43], decoherence is a particular case of the phenomenon of irreversibility, which leads to the following problem. Since the quantum state $\rho(t)$ evolves

unitarily, it cannot follow an irreversible evolution. Therefore, if the non-unitary evolution is to be accounted for, a further element has to be added; precisely, the splitting of the maximal information about the system into a relevant part and an irrelevant part: whereas the irrelevant part is discarded, the relevant part may evolve non-unitarily. This idea can be rephrased in operators language. The maximal information about the system is given by the set $\mathcal{O}$ of all its possible observables. Therefore, by selecting a subset $\mathcal{O}^R \subset \mathcal{O}$, we restrict that maximal information to a relevant part.

Since decoherence is an irreversible process, the splitting of the whole set $\mathcal{O}$ of observables is also required. On this basis, the phenomenon of decoherence can be explained in three general steps:

Step 1. The set $\mathcal{O}^R \subset \mathcal{O}$ of relevant observables is defined.
Step 2. The expectation value $\langle O^R \rangle_{\rho(t)}$ is computed, for any $O^R \in \mathcal{O}^R$.
Step 3. It is proved that $\langle O^R \rangle_{\rho(t)}$ rapidly approaches a value $\langle O^R \rangle_{\rho_*}$ (or that $\rho(t)$ weakly approaches a final state ρ_*).

Since always a coarse-grained state $\rho_G(t)$ can be defined, such that $\langle O^R \rangle_{\rho(t)} = \langle O^R \rangle_{\rho_G(t)}$, the non-unitary evolution of $\rho_G(t)$ (governed by a master equation) can be obtained: $\rho_G(t)$ will rapidly converge to a final state ρ_{G*}, which is obviously diagonal in its own eigenbasis:

$$\langle O^R \rangle_{\rho(t)} = \langle O^R \rangle_{\rho_G(t)} \longrightarrow \langle O^R \rangle_{\rho_*} = \langle O^R \rangle_{\rho_{G*}} \tag{1}$$

This means that, although the off-diagonal terms of $\rho(t)$ never vanish through the unitary evolution, decoherence obtains because it is a *coarse-grained process*: the system decoheres *from the observational point of view* given by any relevant observable $O^R \in \mathcal{O}^R$.

3. Open and closed systems

The need of selecting a set $\mathcal{O}^R$ of relevant observables, in terms of which the time-evolution of the system is described, is explicitly or implicitly admitted by the different approaches to the emergence of classicality: gross observables in van Kampen [1], macroscopic observables of the apparatus in Daneri *et al* [4], observables of the open system in EID [7, 11], collective observables in Omnès [44, 45], van Hove observables in SID [35, 37], etc. It is quite clear that a closed system can be 'partitioned' into many different ways and, thus, there is not a single set of relevant observables essentially privileged (see [46, 47]). Nevertheless, when the emergence of classicality has to be accounted for, certain sets $\mathcal{O}^R$ prove to be physically relevant, in the sense that the expectation values $\langle O^R \rangle_{\rho(t)}$ follow the non-unitary evolution required in step 3, under particular definite conditions that have to be defined in each particular case.

Here we will analyze two ways of selecting the relevant observables, those proposed by the EID and the SID approaches, and we will study the conditions that lead to decoherence in each case. In particular, we will show that both approaches are not conflicting views, but they can be subsumed under the general framework sketched in the previous section.

3.1. Open systems

EID is usually conceived as an open-system approach because it partitions a closed system U into a proper system S and its environment E. However, as we will see, this approach can be rephrased from the viewpoint of the system U in the general framework introduced in the previous section.

Let us consider the Hilbert space $\mathcal{H}$ of the system U, $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$, where $\mathcal{H}_S$ is the Hilbert space of S and $\mathcal{H}_E$ is the Hilbert space of E. The corresponding von Neumann–Liouville space of U is $\mathcal{L} = \mathcal{H} \otimes \mathcal{H} = \mathcal{L}_S \otimes \mathcal{L}_E$, where $\mathcal{L}_S = \mathcal{H}_S \otimes \mathcal{H}_S$ and $\mathcal{L}_E = \mathcal{H}_E \otimes \mathcal{H}_E$.

Step 1. In the case of EID, the relevant observables are those corresponding to the open system S:

$$O^R = O_S \otimes I_E \in \mathcal{O}^R \subset \mathcal{L} \tag{2}$$

where $O_S \in \mathcal{L}_S$ and I_E is the identity operator in $\mathcal{L}_E$.

Step 2. The expectation value of any $O^R \in \mathcal{O}^R$ in the state $\rho(t)$ of U can be computed as

$$\langle O^R \rangle_{\rho(t)} = \mathrm{Tr}(\rho(t)(O_S \otimes I_E)) = \mathrm{Tr}(\rho_S(t) O_S) = \langle O_S \rangle_{\rho_S(t)} \tag{3}$$

where $\rho_S(t) = \mathrm{Tr}_E \rho(t)$ is the reduced density operator of S, obtained by tracing over the environmental degrees of freedom.

Step 3. The EID approach studies the time evolution of $\rho_S(t)$ governed by an effective master equation; it proves that, under certain definite conditions, $\rho_S(t)$ converges to a stable state ρ_{S*}:

$$\rho_S(t) \longrightarrow \rho_{S*}, \qquad \text{then} \quad \langle O^R \rangle_{\rho(t)} = \langle O_S \rangle_{\rho_S(t)} \longrightarrow \langle O_S \rangle_{\rho_{S*}} = \langle O^R \rangle_{\rho_*} \tag{4}$$

where ρ_{S*} is obviously diagonal in its own eigenbasis. This process of convergence of $\rho_S(t)$ to a final stable case has a characteristic time t_{RS}, called *relaxation time of S*, that is, the time that the system needs to reach a state very close to the decohered equilibrium state.

In the EID approach, another relevant time can be defined. Let us consider the following states:

$$|\Psi(0)\rangle = |\psi_S(0)\rangle |\psi_E(0)\rangle, \qquad |\psi_S(0)\rangle = \sum_i a_i |\psi_{iS}(0)\rangle \tag{5}$$

where $|\psi_S(0)\rangle$ is the initial state of the proper system, $|\psi_E(0)\rangle$ is the initial state of the environment and $\{|\psi_{iS}(0)\rangle\}$ is an initial basis of the system such that $|\psi_{iS}(0)\rangle$ are macroscopically distinguishable to each other (for a condition of macroscopic distinguishability, see [44, 45]). In many models it can be shown that

$$|\Psi(t)\rangle = \sum_i a_i |\psi_{iS}(t)\rangle |\mathcal{E}_i(t)\rangle \tag{6}$$

where $|\mathcal{E}_i(t)\rangle$ are the non-orthonormal states of the environment. If the degrees of freedom of the environment are traced over, we obtain the reduced state of the system S:

$$\rho_S(t) = \mathrm{Tr}_E |\Psi(t)\rangle\langle\Psi(t)| = \sum_{ij} a_i \overline{a_j} |\psi_{iS}(t)\rangle\langle\psi_{jS}(t)| \langle \mathcal{E}_j(t)|\mathcal{E}_i(t)\rangle. \tag{7}$$

In different examples it can be proved that, when the environment has many degrees of freedom, for $t \to \infty$, the states of the environment approach orthogonality, $\langle \mathcal{E}_j(t)|\mathcal{E}_i(t)\rangle \to \delta_{ji}$, and $\rho_S(t)$ becomes diagonal in the so-called *moving pointer* basis $\{|\psi_{iS}(t)\rangle\}$. The characteristic time of this process is the *decoherence time* t_{DS} *of S*, which turns out to be extremely short. Therefore, for $t \gg t_{DS}$, $\rho_S(t)$ results

$$\rho_S(t) = \sum_i |a_i|^2 |\psi_{iS}(t)\rangle\langle\psi_{iS}(t)| \tag{8}$$

and follows its non-unitary evolution as a diagonal state, up to reach the final stable state ρ_{S*}.

3.2. Closed systems

SID can be conceived as a closed-system approach because it selects the relevant observables without partitioning the closed system U, but in terms of a different criterion.

Let us consider a quantum system endowed with a Hamiltonian H with continuous spectrum: $H|\omega\rangle = \omega|\omega\rangle$, $\omega \in [0, \infty)$.

Step 1. A generic observable of the system reads

$$O = \int_0^\infty \int_0^\infty \widetilde{O}(\omega, \omega')|\omega\rangle\langle\omega'| \, \mathrm{d}\omega \, \mathrm{d}\omega' \tag{9}$$

where $\widetilde{O}(\omega, \omega')$ is any distribution. The relevant observables O^R are those whose components are given by

$$\widetilde{O^R}(\omega, \omega') = O(\omega)\delta(\omega - \omega') + O(\omega, \omega') \tag{10}$$

where $O(\omega, \omega')$ is a regular function. Then, these relevant observables O^R belong to $\mathcal{O}_{\mathrm{VH}} = \mathcal{O}^R$, which we have called van Hove space (see [2, 3]), and they read

$$O^R = \int_0^\infty O(\omega)|\omega) \, \mathrm{d}\omega + \int_0^\infty \int_0^\infty O(\omega, \omega')|\omega, \omega') \, \mathrm{d}\omega \, \mathrm{d}\omega' \tag{11}$$

where $|\omega) = |\omega\rangle\langle\omega|$, $|\omega, \omega') = |\omega\rangle\langle\omega'|$, and $\{|\omega), |\omega, \omega')\}$ is a basis of $\mathcal{O}_{\mathrm{VH}}$. In turn, states are represented by linear functionals belonging to $\mathcal{O}'_{\mathrm{VH}}$, the dual of $\mathcal{O}_{\mathrm{VH}}$, and they read

$$\rho = \int_0^\infty \rho(\omega)(\omega| \, \mathrm{d}\omega + \int_0^\infty \int_0^\infty \rho(\omega, \omega')(\omega, \omega'| \, \mathrm{d}\omega \, \mathrm{d}\omega' \tag{12}$$

where $\{(\omega|, (\omega, \omega'|\}$ is the basis of $\mathcal{O}'_{\mathrm{VH}}$, that is, the cobasis of $\{|\omega), |\omega, \omega')\}$. Under the usual requirements ($\rho(\omega)$ real, positive and normalized), ρ belongs to a convex space $\mathcal{S} \subset \mathcal{O}'_{\mathrm{VH}}$.

Step 2. The expectation value of any observable $O^R \in \mathcal{O}_{\mathrm{VH}}$ in the state $\rho(t) \in \mathcal{S}$ can be computed as (see [29–33]):

$$\langle O^R \rangle_{\rho(t)} = \int_0^\infty \overline{\rho(\omega)} O(\omega) \, \mathrm{d}\omega + \int_0^\infty \int_0^\infty \overline{\rho(\omega, \omega')} O(\omega, \omega') \, \mathrm{e}^{\mathrm{i}\frac{\omega - \omega'}{\hbar} t} \, \mathrm{d}\omega \, \mathrm{d}\omega'. \tag{13}$$

Step 3. When the function $\overline{\rho(\omega, \omega')} O(\omega, \omega')$ is regular (precisely, when it is L_1 in variable $\nu = \omega - \omega'$), the Riemann–Lebesgue theorem can be applied to equation (13). As a consequence, the second term vanishes and $\langle O^R \rangle_{\rho(t)}$ converges to a stable value:

$$\langle O^R \rangle_{\rho(t)} \longrightarrow \langle O^R \rangle_{\rho_*} = \int_0^\infty \overline{\rho(\omega)} O(\omega) \, \mathrm{d}\omega \tag{14}$$

where ρ_* is diagonal in the eigenbasis of the Hamiltonian. The characteristic time of this process is the *relaxation time t_{RU} of the whole system U*, that is, the time that the system needs to reach a state very close to the decohered equilibrium state.

Since in this case the whole system U is not partitioned into S and E, the concept of moving pointer basis, defined in relation to a basis $\{|\mathcal{E}_i(t)\rangle\}$ of the environment, finds no conceptual meaning in the SID approach.

4. Conditions for decoherence

4.1. Open systems

In the context of the EID approach, the fast convergence of $\rho_S(t)$ has been obtained in several models. The paradigmatic example is the case of a two-state system S strongly coupled

with an environment E composed of a large number N of non-interacting particles. In this situation, the environment behaves as a system with states $|\mathcal{E}_i(t)\rangle$, and it can be proved that, when $N \to \infty$, $\langle\mathcal{E}_i(t)|\mathcal{E}_j(t)\rangle \to 0$ for $t \to \infty$: the environmental states approach orthogonality and, as a consequence, $\rho_S(t)$ approaches diagonality in the moving pointer basis in an extremely short decoherence time t_{DS}.

From the analysis of the models studied with this theoretical framework, it can be concluded that environment-induced decoherence requires: (i) a significant interaction between S and E, and (ii) an environment E with a huge number of degrees of freedom. Many physically relevant models fulfill these conditions: in these cases the emergence of classical behavior can be explained by the EID approach.

4.2. Closed systems

On the basis of the theoretical account of the SID approach, it is clear that self-induced decoherence strictly obtains when the Hamiltonian has a continuous spectrum. Nevertheless, the process also leads to decoherence in quasi-continuous models, that is, discrete models where (i) the energy spectrum is quasi-continuous, i.e., has a small discrete energy spacing, and (ii) the functions of energy used in the formalism are such that the sums in which they are involved can be approximated by Riemann integrals. This condition is rather weak: the overwhelming majority of the physical models studied in the literature on dynamics, thermodynamics, quantum mechanics and quantum field theory are quasi-continuous, and the well-known strategy for transforming sums in integrals is applied.

It is interesting to note that the selection of the relevant observables in SID is also a very weak restriction. In fact, the observables not belonging to the space $\mathcal{O}_{\mathrm{VH}}$ are not experimentally accessible and, for this reason, in practice they are always approximated, with the desired precision, by regular observables for which the approach works satisfactorily (for a full argument, see [15]).

5. Relaxation and decoherence times

Up to this point we have considered the convergence of the expectation values to their final values. Now we will consider the characteristic times involved in the processes.

5.1. Open systems

In several models studied by the EID approach, the decoherence time t_{DS} of an open subsystem S in interaction with its environment E turns out to be the relaxation time t_{RS} of the system S multiplied by a macroscopicity coefficient M:

$$t_{DS} = M t_{RS}. \tag{15}$$

For instance, in equation (47) of [10] or in equation (3.136) of [49], $M = \left(\frac{\lambda_{\mathrm{DB}}}{L_0}\right)^2$, where λ_{DB} is the de Broglie length and L_0 is a macroscopic characteristic length. In turn, in [10, p 51], $M = \left(\frac{\Delta x}{2L_0}\right)^2$, where $\frac{\Delta x}{2L_0}$ is the ratio between a microscopic and a macroscopic characteristic length.

Of course, the interaction between S and E is necessary to obtain a finite relaxation time t_{RS} since, with no interaction, S and E are free evolving systems and t_{RS} is infinite: in this case t_{DS} is also infinite and decoherence is only nominal. In the physically relevant models studied by the EID approach, t_{RS} is finite; thus, t_{DS} is extremely short since the macroscopicity ratios $\frac{\lambda_{\mathrm{DB}}}{L_0}$ or $\frac{\Delta x}{2L_0}$ are extremely small (e.g. 10^{-20}, see [10]). Therefore, $t_{DS} \ll t_{RS}$.

5.2. Closed systems

According to the SID approach, decoherence is an irreversible evolution that decays as $e^{-\frac{\gamma}{\hbar}t}$, where γ is the imaginary part of the pole closer to the real axis of the Hamiltonian resolvent:

$$t_{RU} = \frac{\hbar}{\gamma}. \tag{16}$$

Of course, if the Hamiltonian has no poles, the closed system U behaves as a free evolving system. In this case, the system does not approach to a final equilibrium state: its relaxation time t_{RU} is infinite and, *a fortiori*, decoherence is only nominal.

When the interactions introduce poles in the Hamiltonian, it can be expressed as $H = H_0 + V$, where H_0 is the free Hamiltonian and V is the interaction Hamiltonian containing the poles. It can be proved (see [35]) that, in physically relevant cases, $\gamma \sim V$ and, therefore, $t_{RU} \sim \hbar/V$.

For microscopic systems, V can be estimated of the order of 1 eV (a natural energy scale for quantum atomic interactions, see e.g. [50]); then, the relaxation time t_{RU} is very short, $\sim 10^{-15}$ s (see [35]). For macroscopic systems, V can be computed as $V = NV_i$, where N is the number of subsystems (particles) of U, and each V_i is the interaction between each subsystem and the rest of subsystems; in this case, $t_{RU} \sim \hbar/NV_i$. If we consider again that all V_i are of the order of 1 eV, and that $N = 10^{24}$ (a macroscopic body of 1 mol), the relaxation time t_{RU} is fantastically short, $\sim 10^{-39}$ s.

5.3. Comparing both results

In order to compare the results obtained in both cases, we have to partition the whole system U studied by SID into an open system S and an environment E. Now the Hamiltonian has to be expressed as

$$H = H_0 + V = H_0 + V_{SE} + V_E \tag{17}$$

where V_{SE} represents the interaction between S and E, and V_E represents the interactions of the subsystems (particles) of E among themselves.

If both V_{SE} and V_E contribute with poles, the relaxation time t_{RU} of the whole system U will be computed with γ corresponding to the pole closer to the real axis. Of course, if one of the interactions is zero, the corresponding V has no complex poles (can be conceived as having a pole on the real axis), and the corresponding γ is zero. Therefore, in this case the time t_{RU}, which has to be computed with the pole closer to the real axis, is infinite: as expected, with no interactions in a subsystem of the whole system U, the relaxation of U is only nominal and, as a consequence, its decoherence is also only nominal.

An interesting situation is the case where both interactions have very different strengths, in particular, $V_{SE} \gg V_E$. In this case, a two-times evolution can be described (see [35] for details):

(1) Since $V_{SE} \gg V_E$, in a first step we can neglect V_E and consider the Hamiltonian $H^{(1)} = H_0 + V_{SE}$. With this Hamiltonian we can compute a relaxation time $t_R^{(1)} = \hbar/\gamma_{SE}$, where γ_{SE} is the imaginary part of the pole (or of the pole closer to the real axis) of V_{SE}. If the formalism of SID is applied to this case, for times $t \gg t_R^{(1)}$, the state $\rho_*^{(1)}$ so obtained can be considered diagonal for all practical purposes.

(2) But since the whole system has not reached its final equilibrium state yet, after the first period where V_{SE} is dominant, for times $t \gg t_R^{(1)}$, V_E becomes relevant. In this situation, the total Hamiltonian will be $H^{(2)} = H^{(1)} + V_E$, and the relaxation time $t_R^{(2)} = \hbar/\gamma_E$ can be computed, where γ_E is the imaginary part of the pole (or of the pole closer to the real

axis) of V_E. Again, if the formalism of SID is applied, for times $t \gg t_R^{(2)}$, we will obtain the state $\rho_* = \rho_*^{(2)}$, now completely diagonal.

From this two-times evolution, we can see that:

- Since $\gamma_{SE} \sim V_{SE}$ and $\gamma_E \sim V_E$, when $V_{SE} \gg V_E, \gamma_E$ is the pole closer to the real axis by means of which the relaxation time t_{RU} of the whole system U has to be computed. Therefore, $t_{RU} = t_R^{(2)}$.
- Since $t_R^{(1)}$ is computed only in terms of the interaction between S and E, it can be conceived as the relaxation time t_{RS} of the open system S: $t_{RS} = t_R^{(1)}$.
- Since $V_{SE} \gg V_E$, then $t_{RS} = t_R^{(1)} \ll t_R^{(2)} = t_{RU}$: as expected, the relaxation time t_{RU} of the whole system $U = S \cup E$ will be much longer than the relaxation time t_{RS} of the open system S: $t_{RU} \gg t_{RS}$. In other words, the time that a whole system needs to reach the decohered state of equilibrium is much longer than the time needed by a small subsystem strongly coupled with the rest of the degrees of freedom. In turn, the relaxation time t_{RS} that the system S needs to reach the decohered equilibrium is much longer than its decoherence time t_{DS}, that is, the time at which the state of S becomes diagonal in the moving pointer basis $t_{RS} \gg t_{DS}$. As a consequence, $t_{RU} \gg t_{RS} \gg t_{DS}$.
- It is worth noting that the system S may decohere and relax even in the case that the subsystems of E do not interact to each other. In this case, the interaction V_E is zero and, as explained above, $t_R^{(2)} = t_{RU}$ is infinite: the whole system U does not relax to an equilibrium state and, as a consequence, it does not decohere. Nevertheless, the relaxation time $t_R^{(1)} = t_{RS}$ can still be computed and, for a strong interaction between S and E, it will be extremely short. In turn, the system S may decohere in a decoherence time $t_{DS} \ll t_{RS}$. This means that, even when the whole composite system does not decohere, one of its subsystems strongly coupled with the remaining degrees of freedom may decohere extremely fast.

6. The spin-bath model

The spin-bath model is a very simple model that has been exactly solved in previous papers (see [8]). Let us consider that the system S_0 is a spin-1/2 particle with states $|0\rangle$ and $|1\rangle$. The environment E is composed of N spin-1/2 particles S_i with states $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$. The self-Hamiltonians of S_0 and E are taken to be zero, and S_0 interacts with E via the interaction Hamiltonian H_{SE}:

$$H_{SE} = \frac{1}{2}(|0\rangle\langle 0| - |1\rangle\langle 1|) \sum_{i=1}^{N} g_i (|\uparrow_i\rangle\langle\uparrow_i| - |\downarrow_i\rangle\langle\downarrow_i|) \bigotimes_{j\neq i}^{N} I_j \tag{18}$$

where I_j is the identity operator corresponding to the particle S_j. Then, the total Hamiltonian is simply $H = H_{SE}$.

A pure state of $U = S_0 \cup E$ can be written as

$$|\psi_0\rangle = (a|0\rangle + b|1\rangle) \bigotimes_{i=1}^{N} (\alpha_i|\uparrow_i\rangle + \beta_i|\downarrow_i\rangle) \tag{19}$$

where α_i and β_i are aleatory coefficients such that $|\alpha_i|^2 + |\beta_i|^2 = 1$. Under the action of $H = H_{SE}$, the state $|\psi_0\rangle$ evolves into

$$|\psi(t)\rangle = a|0\rangle|\mathcal{E}_0(t)\rangle + b|1\rangle|\mathcal{E}_1(t)\rangle \tag{20}$$

where

$$|\mathcal{E}_0(t)\rangle = |\mathcal{E}_1(-t)\rangle = \bigotimes_{i=1}^{N} \left(\alpha_i \, \mathrm{e}^{\mathrm{i}g_i t/2}|\uparrow_i\rangle + \beta_i \, \mathrm{e}^{-\mathrm{i}g_i t/2}|\downarrow_i\rangle\right) \tag{21}$$

and the corresponding density matrix will be $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$.

An observable $O \in \mathcal{O}$ of the composite system $U = S_0 \cup E$ can be expressed as

$$\begin{aligned} O = (s_{00}|0\rangle\langle 0| + s_{01}|0\rangle\langle 1| + s_{10}|1\rangle\langle 0| + s_{11}|1\rangle\langle 1|) \bigotimes_{i=1}^{N} \big(&\epsilon^{(i)}_{\uparrow\uparrow}|\uparrow_i\rangle\langle\uparrow_i| \\ &+ \epsilon^{(i)}_{\downarrow\downarrow}|\downarrow_i\rangle\langle\downarrow_i| + \epsilon^{(i)}_{\downarrow\uparrow}|\downarrow_i\rangle\langle\uparrow_i| + \epsilon^{(i)}_{\uparrow\downarrow}|\uparrow_i\rangle\langle\downarrow_i|\big) \end{aligned} \tag{22}$$

where $s_{00}, s_{11}, \epsilon^{(i)}_{\uparrow\uparrow}, \epsilon^{(i)}_{\downarrow\downarrow}$ are real numbers and $s_{01} = \overline{s_{10}}, \epsilon^{(i)}_{\uparrow\downarrow} = \overline{\epsilon^{(i)}_{\downarrow\uparrow}}$ are complex numbers. Then, the expectation value of O in the state $|\psi(t)\rangle$ reads

$$\langle O\rangle_{\psi(t)} = (|a|^2 s_{00} + |b|^2 s_{11})\Gamma_0(t) + 2\,\mathrm{Re}[a\bar{b}s_{10}\Gamma_1(t)] \tag{23}$$

where

$$\Gamma_0(t) = \prod_{i=1}^{N}\left[|\alpha_i|^2\epsilon^{(i)}_{\uparrow\uparrow} + |\beta_i|^2\epsilon^{(i)}_{\downarrow\downarrow} + \overline{\alpha_i}\beta_i\epsilon^{(i)}_{\uparrow\downarrow}\,\mathrm{e}^{-\mathrm{i}g_i t} + \overline{\left(\overline{\alpha_i}\beta_i\epsilon^{(i)}_{\uparrow\downarrow}\right)}\,\mathrm{e}^{\mathrm{i}g_i t}\right] \tag{24}$$

$$\Gamma_1(t) = \prod_{i=1}^{N}\left[|\alpha_i|^2\epsilon^{(i)}_{\uparrow\uparrow}\,\mathrm{e}^{\mathrm{i}g_i t} + |\beta_i|^2\epsilon^{(i)}_{\downarrow\downarrow}\,\mathrm{e}^{-\mathrm{i}g_i t} + \overline{\alpha_i}\beta_i\epsilon^{(i)}_{\uparrow\downarrow} + \overline{\left(\overline{\alpha_i}\beta_i\epsilon^{(i)}_{\uparrow\downarrow}\right)}\right]. \tag{25}$$

6.1. The open-system viewpoint

The open-system viewpoint consists of considering one of the spin-1/2 particles as the proper system and the remaining particles as the environment. Let us consider two cases.

Case a. In the typical situation studied by the EID approach, the proper system is S_0. Therefore, the relevant observables $O^R = O^{S_0}$ are obtained by making $\epsilon^{(i)}_{\uparrow\uparrow} = \epsilon^{(i)}_{\downarrow\downarrow} = 1, \epsilon^{(i)}_{\uparrow\downarrow} = 0$:

$$O^{S_0} = (s_{00}|0\rangle\langle 0| + s_{01}|0\rangle\langle 1| + s_{10}|1\rangle\langle 0| + s_{11}|1\rangle\langle 1|) \bigotimes_{i=1}^{N} I_i. \tag{26}$$

The expectation value of these observables is given by

$$\langle O^{S_0}\rangle_{\psi(t)} = |a|^2 s_{00} + |b|^2 s_{11} + \mathrm{Re}[a\bar{b}s_{10}r(t)] \tag{27}$$

where $r(t) = \langle\mathcal{E}_1(t)|\mathcal{E}_0(t)\rangle$ and

$$|r(t)|^2 = \prod_{i=1}^{N}(|\alpha_i|^4 + |\beta_i|^4 + 2|\alpha_i|^2|\beta_i|^2\cos 2g_i t). \tag{28}$$

Numerical results show that (see [41]), as N increases, $|r(t)|$ quickly decays by several orders of magnitude. This means that the interference is suppressed from the viewpoint of the observables that 'observe' only the spin system S_0.

Case b. Nevertheless, we can also decide to select the observables O^{S_j} that 'observe' just *one* spin system S_j of E as the relevant observables:

$$O^{S_j} = I_{S_0} \otimes O_j \bigotimes_{i\neq j} I_{S_i} \tag{29}$$

where $\epsilon^{(j)}_{\uparrow\uparrow}, \epsilon^{(j)}_{\downarrow\downarrow}, \epsilon^{(j)}_{\uparrow\downarrow}$ are now generic. The expectation value of these observables is given by

$$\langle O^{S_j}\rangle_{\psi(t)} = \langle\psi(t)|O_{S_j}|\psi(t)\rangle = |a|^2(|\alpha_j|^2\epsilon^{(j)}_{\uparrow\uparrow} + |\beta_j|^2\epsilon^{(j)}_{\downarrow\downarrow} + \alpha_j\overline{\beta_j}\epsilon^{(j)}_{\uparrow\downarrow}\,\mathrm{e}^{-\mathrm{i}g_j t} + \overline{\alpha_j}\beta_j\epsilon^{(j)}_{\downarrow\uparrow}\,\mathrm{e}^{\mathrm{i}g_j t}) + |b|^2(|\alpha_j|^2\epsilon^{(j)}_{\uparrow\uparrow} + |\beta_j|^2\epsilon^{(j)}_{\downarrow\downarrow} + \alpha_j\overline{\beta_j}\epsilon^{(j)}_{\uparrow\downarrow}\,\mathrm{e}^{\mathrm{i}g_j t} + \overline{\alpha_j}\beta_j\epsilon^{(j)}_{\downarrow\uparrow}\,\mathrm{e}^{-\mathrm{i}g_j t}). \tag{30}$$

From this equation it is easy to see that $\langle O^{S_j}\rangle_{\psi(t)}$ oscillates and, thus, it has no limit. Therefore, from the viewpoint of the observables that 'observe' only the spin system S_j certainly there is no decoherence. This is not a surprising result when we recall that, in the Hamiltonian of equation (18), the spin systems of the subsystem $E = \bigcup_i S_i$ are uncoupled to each other: each S_i evolves as a free system and, for this reason, E is unable to reach a final stable state.

6.2. The closed-system viewpoint

From the closed-system viewpoint of the SID approach, the system $U = S_0 \cup E$ is considered as a whole. Then, the relevant observables are given by equation (22) and their expectation values are computed as in equation (23).

Numerical results (see [41]) show that, in the general case, the time evolution of equation (23) does not lead to the suppression of the terms in $\langle O\rangle_{\psi(t)}$ that are not diagonal in the energy eigenbasis; as a consequence, the system U does not relax nor decohere from the perspective of SID. Schlosshauer [41] interprets this result as a shortcoming of the SID approach: SID would fail to explain the phenomenon of decoherence in this model, correctly described by the EID approach. According to the author, such a failure is due to the *discrete* nature of the model, even under the condition of quasi-continuous energy spectrum. On this basis, he concludes that SID is likely to fail in other systems composed of discrete subsystems.

When decoherence is understood in the context of the general framework introduced here, it is easy to see that, in spite of the interesting numerical simulations, Schlosshauer's interpretation of the results is misguided, since it relies on comparing processes resulting from different subsets of relevant observables. In fact, in a given system decoherence is not a yes–no phenomenon, but a process relative to the relevant observables chosen for the description. When this point is taken into account, the fact that a system may decohere for a certain subset of relevant observables and may not decohere for a different subset turns out to be natural: in particular, this is what happens in cases (a) and (b) of the previous subsection, both solved from the open-system perspective of EID.

Furthermore, when the relaxation time t_{RU} computed as explained in section 5 is considered, the fact that the whole system U will not decohere from the SID approach, far from being a shortcoming of the approach, is its necessary consequence. Let us recall that t_{RU} is obtained on the basis of the poles of the total Hamiltonian of the closed system U. The spin-bath model is a paradigmatic example of the situation considered in subsection 5.3 and developed in paper [35], where $V_{SE} \gg V_E$. In particular, the model is a case where $V_E = 0$, as explained in that subsection: since the particles of the environment E do not interact to each other, t_{RU} is infinite. Therefore, it can be easily inferred, with no need of numerical simulations, that in this model the whole system U does not decohere. Nonetheless, this is not an obstacle for the decoherence of the subsystem S_0, strongly coupled with the remaining degrees of freedom: the corresponding relaxation time t_{RS_0} can be computed from the interaction Hamiltonian H_{SE}, and the decoherence time t_{DS_0} will be even shorter: $t_{DS_0} \ll t_{RS_0}$.

7. Concluding remarks

In this paper we have proposed a general theoretical framework for decoherence, which encompasses formalisms originally devised to deal just with open or closed systems. When decoherence is understood in this framework, the conceptual difficulties of the EID program turn out to be not as serious as originally supposed. In fact,

- Closed quantum systems may decohere; furthermore, in spite of the fact that EID focuses on open systems, it can also be formulated from the perspective of the whole composite system and, in this case, meaningful relationships between the behavior of the whole system and the behavior of its subsystems can be explained.
- The 'defining systems' problem is simply dissolved by the fact that the splitting of the closed system into an open subsystem and an environment is just a way of selecting the relevant observables of the closed system. Since there are many different sets of relevant observables depending on the observational viewpoint adopted, the same closed system can be decomposed in many different ways: each decomposition represents a decision about which degrees of freedom are relevant and which can be disregarded in any case. Since there is no privileged or 'essential' decomposition, there is no need of an unequivocal criterion to decide where to place the cut between 'the' system and 'the' environment.

From this theoretical perspective, decoherence is not an 'absolute' phenomenon which occurs or does not occur in a given system. In contrast, decoherence is relative to the relevant observables selected in each particular case: there is not a privileged set of relevant observables. The only essential physical fact is that, among all the observational viewpoints that may be adopted to study a quantum system, some of them determine subsets of relevant observables for which the system decoheres.

As a consequence, the formalisms of decoherence for open and closed systems are not rival or alternative, but they cooperate in the understanding of the same physical phenomenon. Therefore, the results obtained in both cases turn out to be relevant: for instance, the large amount of experimental confirmations of EID (see [49]), the complete description of the classical limit of quantum mechanics [31, 32, 36, 38] and the study of the role of complexity in decoherence [34, 37, 39] in the case of SID, and the meaningful relations between the decoherence times computed by EID and the relaxation times computed by SID [35] can all be retained as important acquisitions in the new general framework.

Acknowledgments

We are very grateful to Roland Omnès and Maximilian Schlosshauer for many comments and criticisms. This research was partially supported by grants of the University of Buenos Aires, CONICET, and FONCYT of Argentina.

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