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Self-induced decoherence: a new approach

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Abstract

According to Zurek, decoherence is a process resulting from the interaction between a quantum system and its environment; this process singles out a preferred set of states, usually called "pointer basis", that determines which observables will receive definite values. This means that decoherence leads to a sort of selection which precludes all except a small subset of the states in the Hilbert space of the system from behaving in a classical manner: *environment-induced-superselection—einselection*—is a consequence of the process of decoherence. The aim of this paper is to present a new approach to decoherence, different from the mainstream approach of Zurek and his collaborators. We will argue that this approach offers conceptual advantages over the traditional one when problems of foundations are considered; in particular, from the new perspective, decoherence in closed quantum systems becomes possible and the preferred basis acquires a well founded definition.

Keywords: Quantum mechanics; Decoherence; Closed systems; Pointer basis; Coarse graining; Classicality

1. Introduction

As Bub (1997) says, the theory of decoherence has became the "new orthodoxy" in the quantum physicists community. At present, decoherence is studied and tested in many areas such as atomic physics, quantum optics and condensed matter (see

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references in Paz & Zurek, 2000; Zurek, 2001). In particular, the study of decoherence has acquired a great importance in quantum computation, where the phenomenon of decoherence represents a major obstacle to the implementation of information processing hardware that takes advantage of superpositions. On the other hand, in the field of the philosophy of quantum mechanics, decoherence has been regarded as a relevant element for solving the measurement problem (see e.g. Elby, 1994; Healey, 1995) and for explaining the emergence of the classical macroscopic world (Bacciagaluppi & Hemmo, 1994).

The roots of the decoherence program can be found—though, of course, not under that name—in some papers published in the late 1950s and the early 1960s, whose aim was to describe thermodynamically the process of amplification occurring during measurement: these works showed how the interference terms of a quantum state vanish in the limit of infinite-sized devices and infinite times (see Green, 1957; Daneri, Loinger, & Prosperi, 1962). In the 1970s, the measurement problem was addressed from an open-system perspective in several papers: according to this view, macroscopic systems such as measurement devices are never closed, but interact significantly with their environments (see Zeh, 1970). On the basis of these previous contributions, the theory of decoherence was systematized and developed by Zurek and his collaborators in a great number of works. According to Zurek (1991, 1994), decoherence is a process resulting from the interaction between a quantum system and its environment; this process singles out a preferred set of states, usually called "pointer basis", that determines which observables will receive definite values. This means that decoherence leads to a sort of selection which precludes all except a small subset of the states in the Hilbert space of the system from behaving in a classical manner: *environment-induced-superselection—einselection*—is a consequence of the process of decoherence. In Zurek's terms, arbitrary superpositions are dismissed, and the preferred states become the candidate to classical states: they correspond to the definite readings of the apparatus pointer in quantum measurements, as well as to the points in the phase space of a classical dynamical system. On the basis of this characterization, decoherence would offer an essential ingredient for solving the measurement problem and for explaining the transition from quantum to classical.

As Zurek (1998) points out, the theory of decoherence does not amount to an interpretation of quantum mechanics: in a sense, it provides a necessary complement to some traditional interpretations. Here we will not focus on problems of interpretation. The aim of this paper is to present a new approach to decoherence, different from the mainstream approach of Zurek and his collaborators. We will argue that this approach offers conceptual advantages over the traditional one when problems of foundations are considered; in particular, from the new perspective, decoherence in closed quantum systems becomes possible and the preferred basis acquires a well founded definition.

2. The einselection approach to decoherence

In this section we shall outline the main elements of the traditional approach to decoherence, following the review papers of Paz and Zurek (2000) and Zurek (2001).

For these authors, the first step is to split the universe into the degrees of freedom which are of direct interest to the observer, "the system of interest", and the remaining degrees of freedom usually referred to as "the environment". The environment can be external, such as particles of air or photons scattered off the system, or internal, such as collections of phonons or other internal excitations. Since the system is open, during the decoherence process the interaction with the environment will destroy quantum correlations and will result in a transition from pure states to mixtures which will turn out to be diagonal in the same set of preferred states.

In order to understand Zurek's proposal in formal terms, let us consider a simplified case where the system S is a two-state particle in a pure state (see also d'Espagnat, 1995):

$$|\Psi_S\rangle = \alpha|s_1\rangle + \beta|s_2\rangle. \tag{2.1}$$

The environment E is assumed to be composed of a large number N of spin 1/2 particles that do not interact with one another. At time $t = 0$, the state of the composite system SE is given by:[1]

$$|\Psi_{SE}(0)\rangle = (\alpha|s_1\rangle + \beta|s_2\rangle)\Pi_k(a_k|\uparrow\rangle_k + b_k|\downarrow\rangle_k) \quad k = 1 \text{ to } N. \tag{2.2}$$

where $|\uparrow\rangle_k$ and $|\downarrow\rangle_k$ are the eigenvectors of the z-component of the spin corresponding to the k'th particle. If, for simplicity, it is assumed that the self-Hamiltonians of S and E are zero, the evolution of the system SE is governed by the interaction Hamiltonian H_{SE}; let us also assume that:

$$H_{SE} = (|s_1\rangle\langle s_1| + |s_2\rangle\langle s_2|)\Sigma_k g_k(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)_k. \tag{2.3}$$

Under the influence of this Hamiltonian, the initial state $|\Psi_{SE}(0)\rangle$ evolves into

$$|\Psi_{SE}(t)\rangle = \alpha|s_1\rangle|\varepsilon_1(t)\rangle + \beta|s_2\rangle|\varepsilon_2(t)\rangle, \tag{2.4}$$

where

$$|\varepsilon_1(t)\rangle = \Pi_k(a_k \exp(\mathrm{i}g_k t)|\uparrow\rangle_k + b_k \exp(-\mathrm{i}g_k t)|\downarrow\rangle_k), \tag{2.5}$$

$$|\varepsilon_2(t)\rangle = \Pi_k(a_k \exp(-\mathrm{i}g_k t)|\uparrow\rangle_k + b_k \exp(\mathrm{i}g_k t)|\downarrow\rangle_k). \tag{2.6}$$

In fact, in this case the environment E can be conceived as just another two-state system, with eigenstates $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$, whose interaction with the system S results in the entangled state $|\Psi_{SE}(t)\rangle$. The density operator corresponding to this entangled state will be:

$$\begin{aligned}\rho_{SE}(t) = |\Psi_{SE}(t)\rangle\langle\Psi_{SE}(t)| = {} & |\alpha|^2|s_1\rangle\langle s_1|\,|\varepsilon_1(t)\rangle\langle\varepsilon_1(t)| \\ & + \alpha\beta^*|s_1\rangle\langle s_2|\,|\varepsilon_1(t)\rangle\langle\varepsilon_2(t)| \\ & + \alpha^*\beta|s_2\rangle\langle s_1|\,|\varepsilon_2(t)\rangle\langle\varepsilon_1(t)| + |\beta|^2|s_2\rangle\langle s_2|\,|\varepsilon_2(t)\rangle\langle\varepsilon_2(t)|,\end{aligned} \tag{2.7}$$

where the off-diagonal terms represent the quantum correlations which preclude classicality. According to Zurek, at each time the description of the system S is given

[1] In order to simplify formal expressions, we will omit the symbol $\otimes$ for the tensorial product, as usual in physical literature.

by the *reduced density operator* $\rho_r(t)$, obtained from the total density operator $\rho_{SE}(t)$ by tracing over the environmental degrees of freedom:

$$\rho_r(t) = \mathrm{Tr}_\varepsilon \rho_{SE}(t) = |\alpha|^2 |s_1\rangle\langle s_1| + \alpha\beta^* r(t) |s_1\rangle\langle s_2| + \alpha^*\beta r^*(t) |s_2\rangle\langle s_1| + |\beta|^2 |s_2\rangle\langle s_2|, \tag{2.8}$$

where

$$r(t) = \langle \varepsilon_1(t) | \varepsilon_2(t) \rangle = \Pi_k [\cos 2g_k t + \mathrm{i}(|a_k|^2 - |b_k|^2) \sin 2g_k t] \tag{2.9}$$

determines the relative size of the off-diagonal terms.[2] As time passes, the composite system *SE* evolves under the influence of the interaction Hamiltonian H_{SE} in such a way that $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$ rapidly approach orthogonality:

$$\text{as } t \to \infty, \quad r(t) = \langle \varepsilon_1(t) | \varepsilon_2(t) \rangle \to 0. \tag{2.10}$$

In the infinite-time limit, the reduced density operator turns out to be

$$\rho_r = |\alpha|^2 |s_1\rangle\langle s_1| + |\beta|^2 |s_2\rangle\langle s_2|, \tag{2.11}$$

where the off-diagonal terms have vanished.[3] According to Zurek, ρ_r denotes a mixture which contains only the terms corresponding to classical correlations.

However, this formalism does not yet explain the *process* of decoherence, that is, the dynamics of the quantum open system *S*. As Paz and Zurek (2000) admit, in principle the evolution equation for ρ_r could be obtained by solving the von Neumann equation—the Schrödinger equation for density operators—for the total density operator ρ_{SE} and then taking the partial trace. But this task is analytically very difficult. For this reason, the usual strategy consists in describing the evolution of ρ_r by means of the so-called "*generalized master equation*". The use of partial traces to eliminate environmental variables and to derive generalized master equations was developed by many researchers from the 1950s. Their works added a second term to the von Neumann equation; this term describes the energy dissipation of the open system. By using the path integral formalism, Feynman and Vernon (1963) derived a third term responsible for the fluctuations that lead to Brownian motion, but they were unable to integrate their complex expression. Caldeira and Leggett (1983) were the first authors who derived a closed analytic expression for the third term;[4] in their formalism the density operator exhibits off-diagonal terms that decrease exponentially in time.

The derivation of the generalized master equation depends on the specific features of the system of interest and its environment and, in general, requires to introduce some approximations. For example, by assuming that the system–environment

[2] Unless the *k*th spin of the environment is initially in an eigenstate of the interaction Hamiltonian (in which case the coherence in the system would be retained), its contribution to the product will be less than unity. Therefore, the value of $r(t)$ at a given time decreases with the number *N* of particles of the environment.

[3] The off-diagonal terms of $\rho_r(t)$ approach zero exponentially. The estimates of the exponential decay show that, for realistic experiments, the rates are fantastically fast.

[4] Caldeira and Leggett considered the case of an internal environment. Joos and Zeh (1985) treated the case of the effect of an external environment consisting of particles colliding on a macroscopic body.

interaction is small, the master equation is obtained using perturbation theory: the majority of cases solved in the physical literature has been treated with this technique. The exact master equation has been obtained only in very few cases like, for example, quantum Brownian motion (Paz, 1994): the perturbative results can be shown to be very similar to their exact counterparts. In this case, the derivation of the exact master equation consists in two main steps. The first step is to find the explicit form of the evolution operator, denoted by **J**, of the reduced density operator ρ_r:

$$\rho_r(t) = \mathbf{J}(t, t_0)\rho_r(t_0). \tag{2.12}$$

The second step is to use this explicit form to obtain the master equation satisfied by ρ_r. The explicit form of **J** is obtained by means of a modification of the path integral formalism introduced by Feynman and Vernon. In particular, **J** has a path integral representation of the form

$$\mathbf{J}(x, x', t) = \int Dx \int Dx' \exp(\mathrm{i}2\pi/h)(S(x) - S(x'))F[x, x'], \tag{2.13}$$

where the integrals are over paths satisfying the initial condition (x_0, q_0), $S(x)$ is the action for the system, and $F[x, x']$ is the so-called "influence functional" due to Feynman and Vernon and representing the physical effects produced by the environment on the evolution of the system; in fact, this functional is equal to the identity when there is no interaction between the system and the environment. In the case of Brownian motion, the influence functional is calculated for an environment consisting of a collection of independent harmonic oscillators interacting linearly through position with the system. The influence functional enables to compute the exact expression for the evolution operator **J** and, in turn, with **J** the exact generalized master equation can be obtained.

A central problem that the theory of decoherence must face is the problem of the emergence of the preferred basis. Our experience always shows us the familiar macroscopic objects in some definite state with respect to the usual classical observables such as position or energy. These states are not arbitrary states in the Hilbert space: they appear to be "stable" in the sense that a macroscopic object is always found in one of the states of the same menu of options. Zurek offers a criterion for singling out the preferred set of states, rather than just confirming the suspicion about the classicality of certain observables. He calls this criterion "*predictability sieve*" because it acts as a filter accepting certain states in the Hilbert space of the system and rejecting others. The predictability sieve is based on what Zurek considers the definitional property of the preferred states, namely, their stability: the preferred states are, by definition, the least affected by the interaction with the environment in the sense that they are those that become less entangled with it. According to Zurek, a good measure of the influence of the environment on the system is the observer's ability to predict the future evolution of the system. If, in turn, the entropy of the reduced density operator at time t is taken as a convenient measure of the loss of predictability, then in order to find the preferred states it is necessary to consider all the possible pure states of the system and to compute the

entropy associated with their corresponding reduced density operators after some time t: the preferred states will be those that minimize the entropy production. However, in order to simplify the procedure, instead of using the von Neumann entropy, $h = -\mathrm{Tr}\,\rho_r \log \rho_r$, Zurek studies the evolution of the *purity* of the system as measured by

$$\zeta(t) = \mathrm{Tr}\,\rho_r^2(t). \tag{2.14}$$

This quantity is equal to one for a pure state and decreases when the state becomes mixed. The evolution equation for the purity ζ is obtained by means of the generalized master equation previously computed.

According to Zurek (1991, 2001), this approach to decoherence does not discriminate between the Copenhagen Interpretation and the Many Worlds Interpretation. Rather, it fits into either framework, providing the missing elements of both interpretations: it can supply the definition of the branches of the universal wave function in Everett's approach, and it also delineates the quantum-classical border postulated by Bohr.

3. The new approach to decoherence

In this section we will briefly present a different approach to decoherence by following mainly the papers of Castagnino and Laura (2000a, b). This approach relies on the general idea that the interplay between observables and states is a fundamental element of quantum mechanics (Laura & Castagnino, 1998a; Castagnino, Id Betan, Laura, & Liotta, 2002). As it is well known, the relationship between the state ρ of a closed quantum system and the observable O is expressed by

$$\langle O \rangle_\rho = \mathrm{Tr}(\rho O), \tag{3.1}$$

where $\langle O \rangle_\rho$ is the mean value of the observable O in the state ρ. In the new approach, the process of decoherence can be conceived in the following way. In rough terms,

$$\langle O \rangle_\rho = \Sigma_\omega \Sigma_{\omega'} \rho_{\omega\omega'} O_{\omega\omega'}, \tag{3.2}$$

where $\rho_{\omega\omega'}$ and $O_{\omega\omega'}$ are the components of the operators O and ρ, respectively, in any basis: the $\rho_{\omega\omega'}$ with $\omega \neq \omega'$ represent the quantum correlations which are responsible for the interference terms. Decoherence is the process of transition:

$$\langle O \rangle_\rho = \Sigma_\omega \Sigma_{\omega'} \rho_{\omega\omega'} O_{\omega\omega'} \to \Sigma_\omega \rho_{\omega\omega} O_{\omega\omega} = \langle O \rangle_{\rho *}. \tag{3.3}$$

This means that the effect of decoherence is that the mean value of the observable O is computed, at the end of the process, in a new state represented by the diagonal density operator ρ_*. However, this is a rough presentation because, as we will see, from the new perspective decoherence cannot be explained in a discrete framework to the extent that the phenomenon results from the dynamics of a quantum system governed by a Hamiltonian with continuous spectrum. As a consequence, decoherence does not require the interaction between system and environment:

since the diagonalization of the density operator does not depend on the openness of the system but on the continuous spectrum of the system's Hamiltonian, a single closed system can decohere.

In order to understand how decoherence is explained in this case, it is necessary to point out the theoretical context to which the new approach belongs. In spite of the great success of quantum electrodynamics, in the 1950s the problems related with renormalization led to seek a more rigorous presentation of quantum theory, and a new formulation was developed under the name of "algebraic formalism" by Segal (1947), Wightman (1956), Bogoliubov (Bogoliubov, Logunov, & Todorov, 1975) and Haag (1993), among others. The basic idea consists in adopting an algebra of observables A as the primitive element of the theory; quantum states are represented by linear functionals over A, that is, they belong to a convex set of states $S \subset A'$, where A' is the dual of A. In the original formulation of the algebraic formalism, the algebra of observables is a C^*-algebra. The GNS theorem (Gel'fand–Naimark–Segal) proves that the traditional Hilbert space formalism is a particular representation of this algebraic formalism; the algebra of observables is thereby given a concrete representation as a set of self-adjoint bounded operators on a separable Hilbert space. Nevertheless, it is well known that the C^*-algebraic framework does not admit unbounded operators; however, many important quantum observables, such as position, momentum and energy, usually correspond to operators of this sort. Therefore it is necessary to move to a less restrictive framework in order to accommodate unbounded observable operators. The new approach to decoherence adopts a *nuclear algebra* (see Treves, 1967) as the algebra of observables A: the elements of a nuclear algebra are *nuclei* or *kernels*, that is, two variables distributions that can be thought of as generalized matrices. By means of a generalized version of the GNS theorem (Iguri & Castagnino, 1999), it can be proved that this nuclear formalism has a representation in a *rigged Hilbert space*: the appropriate "rigging" provides a mathematical rigorous foundation to unbounded operators (see Belanger & Thomas, 1990). In fact, the nuclear spectral theorem establishes that, under very general mathematical hypotheses (quite reasonable from a physical point of view), for every CSCO (complete set of commuting observables) of essentially self-adjoint unbounded operators there is a rigged Hilbert space where such a CSCO can be given a generalized eigenvalue decomposition, meaning that a continuum of generalized eigenvalues and eigenvectors may thereby be associated with it.[5] In order to find the appropriate "rigging", the nuclear algebra is used to generate two additional topologies by means of the Nelson operator: one topology corresponds to a *nuclear space* V_0, which is the space of generalized observables; the other topology corresponds to the dual of the space V_0, and this is the space V_S of states. Therefore, unbounded operators, such as position, momentum or energy operators, that have no eigenvalues or eigenvectors in a separable Hilbert space, acquire a continuum of generalized eigenvalues and eigenvectors in the space V_0.

[5] The set of generalized eigenvalues can be extended beyond the spectrum of the associated separable Hilbert space operator to include complex eigenvalues.

A brief remark about terminology: regarding this point, different alternatives still coexist in the literature about the algebraic formalism of quantum theory. Following Antoniou, Suchanecki, Laura, and Tasaki (1997) and Laura and Castagnino (1998a, b), we will symbolize an observable belonging to V_O by a *round ket* $|O)$ and a state belonging to V_S by a *round bra* $(\rho|$. The result of the action of the round bra $(\rho|$ on the round ket $|O)$ is the mean value of the observable $|O)$ in the state $(\rho|$:

$$\langle O \rangle_\rho = (\rho|O). \tag{3.4}$$

If the basis is discrete, $\langle O \rangle_\rho$ can be computed as usual, that is, as the trace of ρO:

$$\langle O \rangle_\rho = (\rho|O) = \mathrm{Tr}(\rho O). \tag{3.5}$$

But if the basis is continuous, $\mathrm{Tr}(\rho O)$ is not well defined; nevertheless, $(\rho|O)$ can always be rigorously defined since, in this case, $(\rho|$ is a linear functional belonging to V_S and acting on an operator $|O)$ belonging to V_O.

In order to see how decoherence works in the new approach, let us consider the simple case of a quantum system whose Hamiltonian has a continuous spectrum

$$|H)|\omega\rangle = \omega|\omega\rangle, \quad \omega \in [0, \infty), \tag{3.6}$$

where ω and $|\omega\rangle$ are the generalized eigenvalues and eigenvectors of $|H)$, respectively. The task is now to express a generic observable $|O)$ in the eigenbasis of the Hamiltonian. The Hamiltonian itself can be expressed as

$$|H) = \int \omega|\omega\rangle\langle\omega| \, \mathrm{d}\omega = \int \omega|\omega) \, \mathrm{d}\omega, \tag{3.7}$$

where $|\omega) = |\omega\rangle\langle\omega|$. Then, all the observables which commute with $|H)$ must have the following form

$$|O) = \int O(\omega)|\omega\rangle\langle\omega| \, \mathrm{d}\omega = \int O(\omega)|\omega) \, \mathrm{d}\omega. \tag{3.8}$$

But, of course, we need also observables which do not commute with H. Therefore, following the formalism of van Hove (1955), a generic observable $|O)$ belonging to the space V_O reads:

$$|O) = \int O(\omega)|\omega) \, \mathrm{d}\omega + \int\int O(\omega, \omega')|\omega; \omega') \, \mathrm{d}\omega \, \mathrm{d}\omega', \tag{3.9}$$

where $O(\omega)$ and $O(\omega, \omega')$ are generic distributions, and $|\omega) = |\omega\rangle\langle\omega|$ and $|\omega; \omega') = |\omega\rangle\langle\omega'|$ are the generalized eigenvectors of the observable $|O)$; $\{|\omega), |\omega; \omega')\}$ is the basis of V_O. On the other hand, states are represented by linear functionals belonging to the space V_S, which is the dual of V_O; therefore, a generic state $(\rho|$ belonging to V_S can be expressed as

$$(\rho| = \int \rho(\omega)(\omega| \, \mathrm{d}\omega + \int\int \rho(\omega, \omega')(\omega; \omega'| \, \mathrm{d}\omega \, \mathrm{d}\omega', \tag{3.10}$$

where $\{(\omega|, (\omega; \omega'|\}$ is the basis of V_S, that is, the cobasis of $\{|\omega), |\omega; \omega')\}$, and it is defined by the following relations:[6]

$$(\omega|\omega') = \delta(\omega - \omega') \quad (\omega; \omega'|\omega''; \omega''') = \delta(\omega - \omega'')\delta(\omega' - \omega''') \quad (\omega|\omega'; \omega'') = 0. \tag{3.11}$$

The only condition that $\rho(\omega)$ and $\rho(\omega, \omega')$ must satisfy is that of leading to a well defined mean value of the observable $|O)$ in the state $(\rho|$:

$$\langle O \rangle_\rho = (\rho|O) = \int \rho(\omega) O(\omega)\, \mathrm{d}\omega + \int\int \rho(\omega, \omega') O(\omega, \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega'. \tag{3.12}$$

This means that $\rho(\omega)$ and $\rho(\omega, \omega')$ must be such that both integrals in Eq. (3.12) are well defined.

With respect to the formalism of van Hove, the new approach introduces two restrictions. The first one consists in considering only the observables $|O)$ whose $O(\omega, \omega')$ are regular functions; these observables define what we will call "*van Hove space*", $V_O^{VH} \subset V_O$. Since the states are represented by linear functionals over the space of observables, in this case they belong to the space $V_O'^{VH}$, which is the dual of the space V_O^{VH}. The second restriction consists in considering only the states $(\rho|$ whose $\rho(\omega, \omega')$ are regular functions; these states belong to a convex space S included in $V_O'^{VH}$: $(\rho| \in S$ and $S \subset V_O'^{VH}$. Under these restrictions, decoherence follows in a straightforward way. According to the unitary von Neumann equation, $(\rho(t)| = e^{-iHt}(\rho_0|e^{iHt}$, the time evolution of the state $(\rho|$ is given by (see Eq. (3.10)):

$$(\rho(t)| = \int \rho(\omega)(\omega|\, \mathrm{d}\omega + \int\int \rho(\omega, \omega') e^{-i(\omega - \omega')t} (\omega; \omega'|\, \mathrm{d}\omega\, \mathrm{d}\omega'. \tag{3.13}$$

Therefore, the mean value of the observable $|O) \in V_O^{VH}$ in the state $(\rho(t)| \in S$ reads:

$$\langle O \rangle_{\rho(t)} = (\rho(t)|O) = \int \rho(\omega) O(\omega)\, \mathrm{d}\omega + \int\int \rho(\omega, \omega') e^{-i(\omega - \omega')t} O(\omega, \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega'. \tag{3.14}$$

Since $O(\omega, \omega')$ and $\rho(\omega, \omega')$ satisfy the condition of being regular functions, when we take the limit for $t \to \infty$, we can apply the *Riemann–Lebesgue theorem*[7] according to which the second term of the right hand side of Eq. (3.14) vanishes. Therefore

$$\lim_{t \to \infty} \langle O \rangle_{\rho(t)} = \lim_{t \to \infty} (\rho(t)|O) = \int \rho(\omega) O(\omega)\, \mathrm{d}\omega. \tag{3.15}$$

But this integral is equivalent to the mean value of the observable $|O)$ in a new state $(\rho_*|$ where the off-diagonal terms have vanished:

$$(\rho_*| = \int \rho(\omega)(\omega|\, \mathrm{d}\omega \Rightarrow \langle O \rangle_{\rho_*} = (\rho_*|O) = \int \rho(\omega) O(\omega)\, \mathrm{d}\omega. \tag{3.16}$$

[6] These are just the generalization of the relationship between the basis $\{|i\rangle\}$ and the cobasis $\{\langle j|\}$ in the discrete case: $\langle j|i\rangle = \delta ij$.

[7] The Riemann–Lebesgue theorem states that: $\lim_{x \to \infty} \int e^{ixy} f(y)\, \mathrm{d}y = 0$ iff $f(y) \in L^1$ (that is, iff $\int |f(y)|\, \mathrm{d}y < \infty$).

Therefore, for all $|O) \in V_O^{VH}$ and for all $(\rho| \in S$, we obtain the limit

$$\lim_{t \to \infty} \langle O \rangle_{\rho(t)} = \langle O \rangle_{\rho_*}. \tag{3.17}$$

This equation represents the definition of decoherence from the new perspective. As we can see, whereas in the einselection approach decoherence is defined as the vanishing of the off-diagonal elements of the system's reduced density operator, here decoherence is characterized in terms of the mean values of observables. In other words, the new definition of decoherence does not involve the evolution of reduced density operators, but the convergence of the mean value of any observable belonging to V_O^{VH} to a value that can be computed as if the system were in a state represented by a diagonal density operator.

Let us now consider the case of a Hamiltonian whose spectrum has a continuous part $0 \leqslant \omega < \infty$, and a discrete part consisting of a single negative value $\omega_1 < 0$. In the basis corresponding to this Hamiltonian, a generic observable $|O) \in V_O^{VH}$ is given by

$$\begin{aligned} |O) = O(\omega_1)|\omega_1) + \int O(\omega_1, \omega)|\omega_1; \omega)\, \mathrm{d}\omega + \int O(\omega, \omega_1)|\omega; \omega_1)\, \mathrm{d}\omega \\ + \int O(\omega)|\omega)\, \mathrm{d}\omega + \int\int O(\omega, \omega')|\omega; \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega', \end{aligned} \tag{3.18}$$

and the mean value of the observable $|O)$ in the state $(\rho(t)|$ reads

$$\begin{aligned} \langle O \rangle_{\rho(t)} = (\rho(t)|O) = \rho(\omega_1)O(\omega_1) + \int \rho(\omega_1, \omega)\mathrm{e}^{-\mathrm{i}(\omega 1 - \omega)t} O(\omega_1, \omega)\, \mathrm{d}\omega \\ + \int \rho(\omega, \omega_1)\mathrm{e}^{-\mathrm{i}(\omega - \omega 1)t} O(\omega, \omega_1)\, \mathrm{d}\omega + \int \rho(\omega)O(\omega)\, \mathrm{d}\omega \\ + \int\int \rho(\omega, \omega')\mathrm{e}^{-\mathrm{i}(\omega - \omega')t} O(\omega, \omega')\, \mathrm{d}\omega\, \mathrm{d}\omega'. \end{aligned} \tag{3.19}$$

In this case the system decoheres since the limit of $\langle O \rangle_{\rho(t)}$ for $t \to \infty$ results:

$$\lim_{t \to \infty} \langle O \rangle_{\rho(t)} = \lim_{t \to \infty} (\rho(t)|O) = \rho(\omega_1)O(\omega_1) + \int \rho(\omega)O(\omega)\, \mathrm{d}\omega = (\rho_*|O), \tag{3.20}$$

where $(\rho_*|$ is the diagonal density operator:

$$(\rho_*| = \rho(\omega_1)(\omega_1| + \int \rho(\omega)(\omega|\, \mathrm{d}\omega. \tag{3.21}$$

However, if the discrete part of the Hamiltonian's spectrum has two or more negative values $\omega_i < 0, i = 1$ to N, the first term of $(\rho(t)|O)$ (Eq. (3.19)) turns out to be

$$\sum_{i,j=1}^{N} \rho(\omega_i, \omega_j)O(\omega_i, \omega_j)\mathrm{e}^{-\mathrm{i}(\omega \mathrm{i} - \omega j)t}. \tag{3.22}$$

The Riemann–Lebesgue theorem does not apply to this sum: in fact, the sum has no limit for $t \to \infty$. This means that, when the discrete part of the Hamiltonian's spectrum has more than one non-overlapping value in its discrete part, the system does not decohere. These considerations serve not only to point out the conditions that the system's Hamiltonian must satisfy in order to lead to

decoherence, but also to show a certain analogy between quantum decoherence and classical mixing, since a classical system is mixing if its Liouville operator has a single non-degenerate eigenvalue zero and the rest of the spectrum is continuous (see Schuster, 1984).

Up to this point we have studied a simplified case where the Hamiltonian was the only dynamical variable. But, in a general case, we must consider a CSCO $\{|H), |O_1), \ldots, |O_n)\}$, whose eigenvectors are $|\omega, o_1, \ldots, o_n\rangle$. In this case, $(\rho_*|$ will be diagonal in the variables ω, ω' but not in general in the remaining variables. Therefore, a further diagonalization of $(\rho_*|$ is necessary: as the result, a new set of eigenvectors $\{|\omega, p_1, \ldots, p_n\rangle\}$, corresponding to a new CSCO $\{|H), |P_1), \ldots, |P_n)\}$, emerges. This set defines the eigenbasis $\{|\omega, p_1, \ldots, p_n), |\omega, p_1, \ldots, p_n; \omega', p'_1, \ldots, p'_n)\}$ of the van Hove space of observables V_O^{VH}, where

$$|\omega, p_1, \ldots, p_n) = |\omega, p_1, \ldots, p_n\rangle\langle\omega, p_1, \ldots, p_n|$$

$$|\omega, p_1, \ldots, p_n; \omega', p'_1, \ldots, p'_n) = |\omega, p_1, \ldots, p_n\rangle\langle\omega', p'_1, \ldots, p'_n| \tag{3.23}$$

$(\rho_*|$ will be completely diagonal in the cobasis of states, $\{(\omega, p_1, \ldots, p_n|, (\omega, p_1, \ldots, p_n; \omega', p'_1, \ldots, p'_n|\}$ corresponding to the new eigenbasis of V_O^{VH} (for details, see Castagnino & Laura, 2000b, Section II-B).

In summary, from the new perspective the process of decoherence depends on the evolution of $\langle O\rangle_{\rho(t)}$, that is, on the interplay between the state $(\rho|$ and the observable $|O)$, and not on the behavior of $(\rho|$ and $|O)$ separately. In fact, even though the new approach is formulated in terms of the Schrödinger picture, the explanation of decoherence could also be given in the Heisenberg picture with few modifications. In this sense, the new account of decoherence is independent of which of the two formalisms is adopted as theoretical background.

4. The physical generality of the new approach

Since the account of decoherence in the new approach is based on the application of the Riemann–Lebesgue theorem, the observables $|O) \in V_O^{VH}$ and the states $(\rho| \in S$ for which decoherence is explained have the following form:

$$|O) = \int O(\omega)|\omega)\, d\omega + \int\int O(\omega, \omega')|\omega; \omega')\, d\omega\, d\omega', \tag{4.1}$$

$$(\rho| = \int \rho(\omega)(\omega|\, d\omega + \int\int \rho(\omega, \omega')(\omega; \omega'|\, d\omega\, d\omega', \tag{4.2}$$

where $O(\omega)$ and $\rho(\omega)$ are generic distributions, and $O(\omega, \omega')$ and $\rho(\omega, \omega')$ are *regular functions*. But, as we have seen, the observables belonging to the van Hove space V_O^{VH} are not all the observables of the system, and the states belonging to the convex space S are not all the functionals of the dual of V_O^{VH}. Then, it is reasonable to ask for an argument to justify the generality of the new approach, that is, an argument to explain in what physical sense the observables belonging to V_O^{VH} and the states belonging to S are general enough to account for decoherence in all physically

relevant situations.[8] As we will see, the generality of the new approach of decoherence relies on the fact that the coordinates of the observables not belonging to V_O^{VH} and of the states not belonging to S in the Hamiltonian's eigenbasis cannot be measured in laboratory and, therefore, they are approximated by their regular counterparts.

Let us suppose that the system's Hamiltonian is

$$|H) = \int \omega|\omega\rangle\langle\omega|\,d\omega, \quad \omega\in[0,\infty). \tag{4.3}$$

For the sake of simplicity, let us consider an observable $|Z) = |z\rangle\langle z|$ such that $\langle z|z\rangle = 1$ (a projector). We know that, in the formalism of van Hove, $|Z)$ can be expressed as

$$|Z) = \int Z(\omega)|\omega)\,d\omega + \int\int\int Z(\omega,\omega')|\omega;\omega')\,d\omega\,d\omega', \tag{4.4}$$

where $Z(\omega)$ and $Z(\omega,\omega')$, representing the diagonal and the off-diagonal coordinates of $|Z)$ in the Hamiltonian's eigenbasis, are generic distributions; this means that, in general, $|Z)$ does not belong to V_O^{VH}. We also know that the off-diagonal coordinates of $|Z)$ in that basis can be computed as

$$Z(\omega,\omega') = \langle\omega|Z|\omega'\rangle = \langle\omega|z\rangle\langle z|\omega'\rangle. \tag{4.5}$$

However, if we want to obtain the specific form of the function $Z(\omega,\omega')$, we have to appeal to a particular density operator. We know that, in a general case

$$\langle O\rangle_\rho = \Sigma_i p_i O_i = \Sigma_i \langle O_i|\rho|O_i\rangle O_i, \tag{4.6}$$

where $p_i = \langle O_i|\rho|O_i\rangle$ is the probability of obtaining the eigenvalue O_i corresponding to the eigenvector $|O_i\rangle$ when the system is in the state $(\rho|$. Since we are working with the observable $|Z) = |z\rangle\langle z|$ whose only non-zero eigenvalue 1 corresponds to the eigenvector $|z\rangle$, its mean value results:

$$\langle Z\rangle_\rho = \langle z|\rho|z\rangle = p. \tag{4.7}$$

Thus, $Z(\omega,\omega')$ can be expressed in terms of the mean value of the observable $|Z)$ in the state $(\rho_{\omega'\omega}| = |\omega'\rangle\langle\omega|$:

$$\langle Z\rangle_{\rho\omega'\omega} = \langle z|\rho_{\omega'\omega}|z\rangle = \langle z|\omega'\rangle\langle\omega|z\rangle = p_{\omega'\omega} = Z(\omega,\omega'), \tag{4.8}$$

where $p_{\omega'\omega}$ is the probability of obtaining the eigenvalue 1 in the state $(\rho_{\omega'\omega}|$. Now let us consider the usual procedure for measuring the function $Z(\omega,\omega') = \langle z|\omega'\rangle\langle\omega|z\rangle$. We divide the $\omega\omega'$-plane in squares $\Delta\omega\Delta\omega$. For each pair $\omega_k\omega_l$ we prepare the system in the state $(\rho_{\omega k\omega l}| = |\omega_k\rangle\langle\omega_l|$ and, after many repetitions of the experiment, we compute $p_{\omega k\omega l} = \langle z|\omega_k\rangle\langle\omega_l|z\rangle$ as the corresponding quotient between the number of times that the eigenvalue 1 is obtained and the total number of repetitions. Finally, when $p_{\omega k\omega l} = \langle z|\omega_k\rangle\langle\omega_l|z\rangle$ is obtained for all the pairs $\omega_k\omega_l$, we construct a regular function $f(\omega,\omega')$ by joining all the obtained values by means of a continuous surface. Of course, $f(\omega,\omega')$ does not represent the

[8] We are grateful to the anonymous referee who drew our attention to the relevance of addressing this point.

off-diagonal coordinates of the observable $|Z)$ in the Hamiltonian's eigenbasis. However, $f(\omega,\omega')$ can be conceived as representing the off-diagonal coordinates of a new observable $|Z_{\mathrm{VH}})$ whose form is

$$|Z_{\mathrm{VH}}) = \int Z(\omega)|\omega)\,\mathrm{d}\omega + \int\int f(\omega,\omega')|\omega;\omega')\,\mathrm{d}\omega\,\mathrm{d}\omega', \tag{4.9}$$

where $f(\omega,\omega')$ is, by construction, a regular function; therefore, $|Z_{\mathrm{VH}})$ belongs to the van Hove space $V_{\mathrm{O}}^{\mathrm{VH}}$. But the central point is that, if $\Delta\omega$ is the minimal energy difference that our instruments can discriminate, no matter how small is such an energy difference, the observable $|Z_{\mathrm{VH}})$ belonging to $V_{\mathrm{O}}^{\mathrm{VH}}$ is observationally indistinguishable from the original observable $|Z)$.

This argument, that has been developed in the case of a projector, can be extended to any generic observable since, according to the spectral decomposition theorem, all observables can be decomposed into a weighted sum of projectors. On the other hand, a similar argument can be constructed in the case of states. In fact, when we measure the off-diagonal coordinates $\rho(\omega,\omega')$ of a generic state $(\rho|$,[9] we obtain a regular function $g(\omega,\omega')$ that can be conceived as representing the off-diagonal coordinates of a new state $(\rho_{\mathrm{VH}}|$ having the same diagonal coordinates as $(\rho|$: $(\rho_{\mathrm{VH}}|$ is observationally indistinguishable from $(\rho|$ and belongs to the space S. These arguments show that the observables not belonging to $V_{\mathrm{O}}^{\mathrm{VH}}$ and the states not belonging to S cannot be experimentally characterized; however, they can always be approximated, with the desired precision, by observables belonging to $V_{\mathrm{O}}^{\mathrm{VH}}$ and by states belonging to S, respectively.

This conclusion serves to argue for the physical generality of the self-induced approach in spite of the fact that it explains decoherence only for observables belonging to $V_{\mathrm{O}}^{\mathrm{VH}} \subset V_{\mathrm{O}}$ and for states belonging to $S \subset V_{\mathrm{O}}'^{\mathrm{VH}}$. The point is that the observables not belonging to $V_{\mathrm{O}}^{\mathrm{VH}}$ and the states not belonging to S are not experimentally accessible; for this reason, physicists usually work with their regular approximations. In more general terms, even though it might be argued that regular functions are mathematically "specific" in the set of all possible distributions, they are not specific in the physical sense given by the possibility of being experimentally characterized. This means that, no matter how small the "size" of $V_{\mathrm{O}}^{\mathrm{VH}}$ is in the set of all the system's observables or the "size" of S is in the set of all the system's states (if there were acceptable definitions of these "sizes", for instance, in terms of measure), the observables not belonging to $V_{\mathrm{O}}^{\mathrm{VH}}$ and the states not belonging to S must always be approximated by regular observables and states, respectively, when quantum theory is tested in laboratory.

Summing up, the new approach explains decoherence only for a subset of all the system's observables and for a subset of all the possible states of the system. Nevertheless, the method has physical generality to the extent that observables and states that are not considered by the formalism are not experimentally accessible

[9] $\rho(\omega,\omega')$ can be measured by means of the mean value of the observables $|O_A) = 1/2(|\omega\rangle\langle\omega'| + |\omega'\rangle\langle\omega|)$ and $|O_B) = 1/2i\,(|\omega\rangle\langle\omega'| - |\omega'\rangle\langle\omega|)$ in the state $(\rho|$: $\langle O_A\rangle_\rho$ gives the real part of $\rho(\omega,\omega')$ and $\langle O_B\rangle_\rho$ gives the coefficient of the imaginary part of $\rho(\omega,\omega')$ (see Ballentine, 1998). Similar procedures can be used to measure $Z(\omega)$ and $\rho(\omega)$, but this point is not relevant for the present argument.

and, for this reason, in practice they are approximated, with the desired precision, by regular observables and states for which the new approach works satisfactorily.

5. General advantages of the new approach

The new approach to decoherence has been applied to different physical situations, showing that its results agree with those obtained by means of the traditional einselection approach. For example, when the new strategy is used for studying models based on the observables P and Q, such as those found in the literature, the results coincide with those obtained by Zurek (1994) and Zurek, Habib, and Paz (1993) (see Castagnino & Laura, 2000b, Appendices A and B). In particular, the characteristic time of decoherence has been computed in the new theoretical context, and the obtained results coincide with the decoherence times obtained in the einselection framework (see Castagnino, Laura, & Id Betan, 1999). The new approach has been also used to prove, in a simpler way, many of the results of the Gell-Mann and Hartle's (1990) "decoherence of histories" theory (see Castagnino & Laura, 2000b, Appendix C). On the other hand, in recent papers this method has been refined in the physical sense as in the mathematical sense. For example, whereas in Castagnino and Laura (2000b) the Hamiltonian of the whole closed system has a spectrum with a continuous section $\omega \in [0, \infty)$ and one non-overlapping negative eigenvalue $\omega_d < 0$ in the discrete section, in Castagnino et al. (1999) and Castagnino, Laura, and Id Betan (2003) the new approach is applied to a situation of the kind treated in the einselection approach: an open system having a Hamiltonian with a discrete spectrum $\omega_i > 0$ interacts with an environment whose Hamiltonian has continuous spectrum; these works show how, as a result of the interaction, the discrete components of the spectrum of the system's Hamiltonian "dissolve" but leave their imprint on the continuous spectrum corresponding to the environment—such as the imprint left by the decaying levels of an atom in the radiation spectrum. In turn, in Castagnino and Ordoñez (2001) the new formalism for decoherence is developed with more mathematical precision as an algebraic formalism in nuclear spaces.

However, all these works are very few when compared with the great number of papers produced from the einselection perspective. So, why should we prefer the new approach over the traditional one as a conceptual basis for decoherence? The reason is that the new approach has relevant advantages from a theoretical point of view.

As we have seen, Zurek's strategy for explaining the process of decoherence relies on the use of the generalized master equation, whose specific form depends on the particular features of the system of interest and the considered environment; this means that the generalized master equation must be construed case by case. Moreover, in general the study of the evolution of the density operator is accomplished by introducing some approximations such as the use of perturbative techniques. In other words, the einselection approach requires this kind of indirect procedures for showing that decoherence occurs. The new approach, on the contrary, does not require all these resources for exhibiting the *theoretical basis* of

decoherence: the process is explained in a completely general way, with no reference to the particular features of the system and the environment, and with no approximations. The conceptual meaning of the phenomenon directly arises from a general mathematical formalism.

On the other hand, the path integral formalism is the mathematical tool necessary for obtaining the generalized master equation; at this point, it is convenient to remember how this kind of formalism works. In strict mathematical terms, a "usual" integral of a function $f(\boldsymbol{x})$, where $\boldsymbol{x} = (x_1, \ldots, x_n)$ is a point in an n-dimensional space, requires the definition of a measure μ on such a space; then, the integral of $f(\boldsymbol{x})$ reads:

$$\int f(\boldsymbol{x})\mu(\mathrm{d}\boldsymbol{x}). \tag{5.1}$$

The Feynman integral is a generalization of the above idea: the point is replaced by a path $g(\boldsymbol{x})$ which connects two fixed points P_1 and P_2. Therefore, the Feynman integral reads:

$$\int F[g(\boldsymbol{x})]D(\boldsymbol{x}), \tag{5.2}$$

where $F[g(\boldsymbol{x})]$ is a functional over the set of paths between P_1 and P_2, and $D(\boldsymbol{x})$ plays the role of $\mu(\mathrm{d}\boldsymbol{x})$. The Feynman integral has been proved to be an extremely useful tool for deriving theorems (e.g., in gauge field theories). However, there is not always a rigorous mathematical way to define $D(\boldsymbol{x})$ as a measure (see Rivers, 1987), and in general the integral (5.2) can only be computed as a limit known as "skeletization" (see Feynman & Hibbs, 1965; Schulman, 1996). In other words, whereas the integral (5.1) can be analytically computed, for instance, by using Barrow's theorem, the integral (5.2) usually can only be computed *numerically* as a limit (this procedure is a generalization of Hartree–Fock computational method). This means that, even in the case of an exact master equation, the results showing that decoherence occurs can only be obtained by means of numerical techniques, usually with the help of computer calculations. Of course, numerical methods are an unavoidable resource in physics for describing particular models: the einselection approach provides many interesting results by following this general strategy. But when foundational matters are considered, numerical methods may hide the conceptual core of the problem at issue. In this sense, the new approach offers the advantage of explaining the mechanism of decoherence by means of a well founded mathematical framework and without appealing to numerical methods: the evolution of the density operator is directly obtained by means of the von Neumann equation, and decoherence results from the application of the well known Riemann–Lebesgue theorem.

As we have seen, in the einselection approach decoherence is usually defined as the vanishing of the off-diagonal elements of the reduced density operator. However, this characterization passes over the crucial distinction between "true" and "false" decoherence (see Unruh, 2000).[10] In fact, there are situations where the correlations

[10] We are grateful to the anonymous referee who drew our attention to this point.

between system and environment lead to the diagonalization of the reduced density operator at a time t, but the interference terms again manifest themselves at later times. This is the case when the system changes slowly with respect to the environment, which then can adjust continually to the changes of the system: the environment adiabatically tracks the system and restores the coherence initially lost. In some situations, the cycle repeats itself endlessly, with the system's purity oscillating between its minimum and maximum values forever. It is quite clear that, if the system recovers its original coherence, decoherence is only apparent since it does not actually suppress quantum interference effects. In the new approach, by contrast, the mere diagonalization of the density operator is not the only requirement for decoherence; the definition of decoherence is given by the limit:

$$\lim_{t \to \infty} \langle O \rangle_{\rho(t)} = \langle O \rangle_{\rho_*}, \tag{5.3}$$

where $(\rho_*|$ is a diagonal density operator. This mathematical limit imposes a stronger condition since it requires that $\langle O \rangle_{\rho(t)}$ asymptotically approaches $\langle O \rangle_{\rho_*}$. In other words, the limit involved in the Riemann–Lebesgue theorem guarantees that, if quantum interference effects are very small at a given time, they become arbitrarily smaller as time goes on.[11] Therefore, the mathematical precise definition of decoherence supplied by the new approach avoids the problems arising from "false" decoherence situations.

The new approach to decoherence also allows us to emphasize another theoretical point. Even if von Neumann's (1955) pioneering work on Hilbert space theory became the mathematical foundation for quantum theory right up to the present, physicists usually preferred to use Dirac's bracket formalism because of its practical convenience; however, Dirac's formalism cannot be given a rigorous mathematical foundation in a separable Hilbert space. At present it is well known that many familiar results of quantum mechanics cannot be obtained in the traditional Hilbert formalism. For instance, Dirac's "delta functions" are special cases of distributions that have not place in a separable Hilbert space, which admits only vectors of finite norm; moreover, many crucial observable operators have continuous spectrum but have no eigenvalues or eigenvectors in a separable Hilbert space. For these reasons, during the last decades many authors have considered that the traditional Hilbert space formulation of quantum mechanics must be extended to a broader mathematical framework when conceptual issues are considered. The new approach to decoherence points to the same conceptual direction to the extent that it adopts an algebraic formalism based on a nuclear algebra, which is better adapted for treating foundational problems. The einselection approach, on the contrary, never abandons

[11] It is also interesting to note that, in all the cases of "false" decoherence explained by Unruh, the interaction Hamiltonian is not constant but changes with time, showing that the composite system, system of interest + environment, is itself open and interacts with something else responsible for the time-dependence of that Hamiltonian. These cases fit in the einselection approach since it deals with open systems. But since the new approach describes decoherence in closed systems, the description of situations like those proposed by Unruh would require the specification of the second environment: the system studied by the new approach would consist of system of interest + environment + second environment, and this whole closed system would have a total Hamiltonian constant with time.

the traditional Hilbert formalism. According to Zurek, "quantum mechanics is formulated in Hilbert space" (Zurek, 1991, p. 41), and the problem of the transition from quantum to classical arises from the fact that

> the set of all states in the Hilbert space is enormous as compared with the size of the set of states where one finds classical systems. (Paz & Zurek, 2000, p. 4)

The new perspective shows the need of transgressing the limits of the separable Hilbert space formalism and of appealing to stronger mathematical tools. The cost of using a broader mathematical framework is compensated by a better conceptual understanding of a crucial phenomenon such as decoherence.

Nevertheless, even though these are important advantages, they are not yet the main theoretical reasons for preferring the new approach to decoherence over the einselection approach. In the next sections we will show that the new approach explains decoherence in closed systems and provides a better founded definition of the preferred basis.

6. Decoherence in closed systems

According to Zurek,

> *decoherence* is a process which—through the interaction of the system with external degrees of freedom often referred as the *environment*—singles out a preferred set of states. (Zurek, 1994, p. 176)

The emergence of the preferred basis results from an environment-induced-superselection which eliminates the vast majority of the "non-classical" states in the Hilbert space: einselected states are distinguished by their stability in spite of the monitoring environment. In Paz and Zurek's words, "the environment distills the classical essence of a quantum system" (Paz & Zurek, 2000, p. 3).

These statements make clear that, from the einselection view, the split of the Universe into the degrees of freedom which are of direct interest to the observer—the system—and the remaining degrees of freedom—the environment—is absolutely essential for decoherence. Such a split is necessary, not only for explaining quantum measurement, but also for understanding "the quantum origin of the classical world" (Paz & Zurek, 2000, p. 1). In fact, Zurek and his collaborators always consider the problem of the transition from quantum to classical as the core of the discussion:

> The aim of the program of decoherence and einselection is to describe the consequences of the 'openness' of quantum systems to their environments and to study the emergence of the effective classicality of some of the quantum states and of the associated observables. (Zurek, 1998, p. 1)

In this context, quantum measurement is conceived as a particular case of the general phenomenon of the emergence of classicality, that is, as an example that illustrates the quantum origin of the classical definiteness of some states in individual

systems (Zurek, 2001). In addition, if classicality only emerges in open quantum systems, it must always be accompanied by other manifestations of openness, such as dissipation of energy into the environment. Zurek even considers that the prejudice which seriously delayed the solution of the problem of the transition from quantum to classical is itself rooted in the fact that the role of the "openness" of a quantum system in the emergence of classicality was ignored for a very long time (Paz & Zurek, 2000; Zurek, 2001).

In summary, decoherence explains the emergence of classicality, but only open systems can "decohere". The question is: what about the Universe as a whole? Zurek himself admits that the Universe is, by definition, a closed quantum system, "it is practically the only system that is effectively closed" (Zurek, 1991, p. 42);[12] but then, the Universe cannot decohere. Zurek considers the possible criticism:

> the Universe as a whole is still a single entity with no 'outside' environment, and, therefore, any resolution involving its division into systems is unacceptable. (Zurek, 1994, p. 181).

It is curious to note that this problem has been pointed out only by few authors (see Pessoa Jr., 1998). In the philosophical literature, decoherence is studied not only in the context of the measurement problem (see Elby, 1994; Bacciagaluppi & Hemmo, 1996; Bub, 1997; Adler, 2003), where the openness of the quantum system cannot be ignored, but also as a relevant element for explaining how "macrofacts" arise (see Bacciagaluppi & Hemmo, 1994; d'Espagnat, 1995): in all cases, the essential role of the interaction between system and environment is unquestionably accepted (see also Healey, 1995).

Even though Zurek recognizes that the quantum state of the Universe as a whole—including the observer, the observed system and the environment—may exist in principle, he considers that this is not the object of study of quantum mechanics (Zurek, 2001). Zurek's answer to the closed-Universe objection is based on his particular conception about the nature of quantum mechanics: for him, the aim of the theory is to establish the relationships between formal results and observer's perceptions. In particular, the problem of the transition from quantum to classical amounts to the question "why don't we perceive superpositions?" (Zurek, 1998, p. 20). This means that the task is to explain, not the emergence of classicality, but *our perception of classicality*:

> the only sensible subject of considerations aimed at the interpretation of quantum theory—that is, at establishing correspondence between the quantum formalism

[12] Of course, "open" and "close" must not be understood in a cosmological sense (that is, in the sense that Robertson-Walker universes with $k = 0$ or -1 are open). Somebody might note that, in certain cases, the time evolution of quantum systems having an infinite number of degrees of freedom becomes problematic and, therefore, such systems cannot be considered closed in the usual dynamical sense. However, in the present context, openness and closeness are characterized on the basis of the interaction between system and environment. Therefore, even those cases with problematic time evolution are closed in this sense when they have no environment to interact with: this is the sense of openness and closeness that is relevant in the present discussion.

> and the events perceived by us—is the relation between the universal state vector and the states of memory (records) of somewhat special systems—such as observers—which are, of necessity, perceiving the Universe from within. (Zurek, 1994, p. 181)

But for Zurek, *perception* and *memory* are not functions of an unphysical "consciousness" responsible for the collapse of the state vector, as in Wigner's interpretation; on the contrary, mental processes correspond to well defined information processing functions which are carried out by physical systems, our brains. In particular, the states of neurons are the seat of memory: neurons are strongly coupled to the environment and are definitely macroscopic enough to behave in an effectively classical way. On the other hand, perception, awareness and other higher functions of the brain are the result of physical interactions which lead to stable correlations between the state of the neurons and the state of the environment, where the state of the perceived object has left its imprint. In other words, the brain is conceived as a massive, neural network-like computer very strongly coupled to its environment, and the environment plays the role of a commonly accessible internet-like data base, which allows the observer to make copies of the records concerning the states of the system with no danger of altering it (Zurek, 1998). The stability of the correlations between the state of the observer's brain and the state of the environment on the one hand, and between the state of the environment and the state of the observed system on the other, is responsible for the perception of classicality. As a result,

> in spite of the undeniable quantum nature of the fundamental physics involved, perception and memory have to rely on the information stored in the decohered (and, therefore, effectively classical) degrees of freedom. (Paz & Zurek, 2000, p. 64)

As we see, in order to answer the closed-Universe objection Zurek appeals to a particular conception of quantum mechanics according to which the aim of the theory is to explain the correspondence between quantum formalism and observer's perceptions, and immerses such a conception into a broader context referred to the nature of perception and memory. But this position would hardly convince the cosmologist, who conceives the Universe as a single closed entity with no other entity to interact with. Quantum cosmology studies the Universe in its early stages, when the quantum effects are not negligible; however, its explanations are considered acceptable only when they lead to the well known results of general relativity in the classical limit. In this context, the wave function of the universe describes, not the system of everything except the observers' brains, but the Universe as a whole. Nevertheless, quantum cosmology tries to explain, with the universal wave function, the evolution of a closed quantum Universe where the classical behavior described by general relativity emerges. If we take Zurek's position seriously, the work of contemporary cosmologists appears as unavoidably hopeless to the extent that, without the assumption of a preexisting division of the Universe into individual systems, the problem of the emergence of classicality has no solution:

> The state of a perfectly isolated fragment of the Universe—of, for that matter, of the quantum universe as a whole—would evolve forever deterministically [...]. The issue of the 'classicality' of its individual components—systems—cannot even be posed. (Zurek, 1994, p. 181).

At this point, somebody could note that the einselection approach has been applied to the cosmological level with interesting results. This is certainly true, but does not undermine the closed-Universe objection. In the works where the einselection approach is used in cosmology, the general strategy consists in splitting the universe into some degrees of freedom which represent the "system" of interest, and the remaining degrees of freedom that are supposed to be non accessible and, therefore, play the role of an internal environment. For instance, in quantum field theory, it is usual to perform a decomposition on a scalar field ϕ, $\phi = \phi_S + \phi_E$, where ϕ_S denotes the system field and ϕ_E denotes the environment field; when it is known that the background field follows a simple classical behavior, the scalar field is decomposed according to $\phi = \phi_c + \phi_q$, where the background field ϕ_c plays the role of the system and the fluctuation field ϕ_q plays the role of the environment (see Calzetta, Hu, & Mazzitelli, 2001). This means that, strictly speaking, it is not the Universe that decoheres, but a subsystem of the Universe: we *perceive* a classical Universe because there are degrees of freedom that act as an environment and to which our brains are correlated.

These considerations allow us to point out the weakest spot of the einselection program. When this approach is applied to the Universe—and, in general, to any system with internal environment—the space of observables which will behave classically must be assumed in advance: the distinction between the system's degrees of freedom and the environmental degrees of freedom is established in such a way that the system decoheres in some observable of this space. This means that the split of the whole must be decided case by case: there is not a general criterion for discriminating between system and environment. In fact, in the case of the decomposition of the scalar field ϕ previously mentioned, different criteria are used: sometimes the decomposition is performed on the basis of the length, mass or momentum scales of the system and the environment; sometimes the system field is considered as containing the lower modes of ϕ and the environment field as containing the higher modes (Calzetta et al., 2001). Zurek recognizes that this lack of a general criterion for deciding where to place the "cut" between system and environment is a serious difficulty of his proposal:

> In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the 'systems' which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best. (Zurek, 1998, p. 122)

The new approach to decoherence overcomes these problems. As the simple case presented in Section 3 shows, decoherence does not require the openness of the system of interest and its interaction with the environment: *a single closed system can*

decohere. If the theoretical formulation is examined, it is not hard to realize that the diagonalization of the density operator does not depend on the openness of the system but on the *continuous spectrum* of the system's Hamiltonian. This feature is precisely what allows us to rigorously apply the Riemann–Lebesgue theorem, which leads to a sort of destructive interference in the off-diagonal terms. In fact, two cases of closed quantum systems have been studied by means of the new approach, showing that these systems decohere with no interaction with an environment. The first case is the so-called "Mott problem": in a bubble chamber, a radiating nucleus endowed with spherical symmetry emits particles whose trajectories develop a pattern not spherically symmetric; the problem consists in explaining how a symmetric spherical structure becomes a non-symmetric radial one. By means of the new approach applied to the system nucleus + bubble chamber, the classical spherically symmetric trajectories are obtained as a result of decoherence (Castagnino & Laura, 2000a). But it is even more interesting that essentially the same equations can be used for describing the evolution of a closed quantum Universe, whose classical behavior arises as a consequence of decoherence: in particular, the resulting diagonal density operator, when translated into a classical density, is resolved as a sum of classical trajectories[13] (Castagnino & Lombardi, 2003). The point here is that *no environment is needed* in order to obtain these results: the assumption that the Universe is a closed quantum system with a *continuous energy spectrum* is the only additional element required for decoherence.

This means that the problem of providing a general criterion for discriminating between system and environment vanishes since decoherence does not require the interaction between them. This fact leads to an additional advantage of the new way of conceiving decoherence. As we have seen, in many cases the einselection approach requires to introduce assumptions about the observables which will behave classically in order to decide where to place the boundary between system and environment. The new approach, on the contrary, provides a mathematically precise definition of the observables regarding to which the system will decohere. As the argument in Section 3 shows, there are two kinds of such observables:

- Observables that commute with the Hamiltonian, whose coordinates in the Hamiltonian's eigenbasis are given by a distribution $O(\omega)$.
- Observables that do not commute with the Hamiltonian, whose diagonal and off-diagonal coordinates in the Hamiltonian's eigenbasis are given by a distribution $O(\omega)$ and a regular function $O(\omega, \omega')$, respectively.

This definition is completely general and does not require to introduce any prior assumption about the classical behavior of certain observables.

When the process of decoherence is viewed from this new perspective, it does not need to be conceived as "a justification for the persistent impression of 'reality'" (Paz & Zurek, 2000, p. 8). Classicality is not a perceptual result of the correlations

[13] In order to perform the calculations, in this case it is necessary to obtain the time t from the Hamiltonian of the Universe that appears in the timeless Wheeler–De Witt equation. This is accomplished by using Hartle's semiclassical limit, the standard method in quantum gravity.

between the observed system and the observer's brain through the environment: the emergence of classicality is a consequence of *the closed system's own dynamics*, to the extent that this dynamics is governed by the Hamiltonian of the system. The transition from quantum to classical does not require the split of the Universe into subsystems as a necessary condition: in contrast to Zurek's assumption, decoherence can also take part in the account of how the Universe as a whole behaves classically. For these reasons we will call decoherence as explained by this new approach "*self-induced decoherence*". In summary, from the new perspective decoherence is a relevant element for explaining the *emergence* of classicality, not *our perception* of classicality.

If the essential ingredient of decoherence is the continuous spectrum of the system's Hamiltonian, it is clear that the theoretical explanation of the decoherence process requires a formalism capable of representing operators with continuous spectrum. But, as we have seen, the einselection approach never abandons the traditional Hilbert space formalism of quantum mechanics with its shortcomings for treating distributions and operators with continuous spectrum. This makes us suspect that the main reason which prevents Zurek and his collaborators from arriving to a general theoretical account of decoherence is the fact that they are using a mathematical tool not adapted for this goal. If the traditional formalism is not replaced by a more powerful mathematical framework, all the results regarding decoherence will necessarily require simplifying assumptions and approximations which hide the very nature of the physical phenomenon.

Nevertheless, it is still legitimate to ask how the einselection approach could arrive to so many results disregarding the essential role of the continuous spectrum of the system's Hamiltonian. When the cases treated in the literature are carefully analyzed, it is not hard to realize that the role of the environment is just to introduce a Hamiltonian with continuous or quasi-continuous spectrum to be coupled with the system of interest. For example, in the simple model of a quantum particle interacting locally with a quantum scalar field which plays the role of the environment (Paz & Zurek, 1998), such a field is what introduces the continuous part in the spectrum of the Hamiltonian of evolution. In the case of decoherence in quantum Brownian motion, as we have seen, the system of interest is a quantum particle immersed into an environment formed by an ensemble of harmonic oscillators interacting linearly through position with the system (Paz, 1994); however, since the distance between the eigenvalues of the environment's Hamiltonian is very small, the results obtained by numerical calculations lead to approximate decoherence, which would become exact decoherence if the environment had a continuous spectrum. In other examples, fluctuations or imperfections of continuous or quasi-continuous nature introduce the continuous part of the spectrum and make the off-diagonal terms of the reduced density operator vanish; this is the case of the spin recombination experiment which takes place in a single crystal interferometer (see Ballentine, 1998). These considerations show why, according to the einselection approach, a simple discrete quantum system never decoheres and it is necessary to immerse it into an environmental bath. When decoherence is understood from the self-induced perspective, the reason is not that

the openness of the system is indispensable, as Zurek claims: the reason lies in the fact that the environment supplies the continuous spectrum necessary for decoherence.

7. The emergence of the preferred basis

As we have seen, according to the einselection approach to decoherence the definitional property of the preferred states is their stability: the preferred states are, by definition, *the least affected by the interaction with the environment*. Zurek's criterion for singling out the preferred basis is the predictability sieve, according to which the preferred states are those that minimize the loss of predictability.

The self-induced approach to decoherence agrees with Zurek's perspective regarding the definition of the preferred states in terms of their stability. However, the emergence of the preferred basis is accounted for in a straightforward way, with no reference to the predictability of the selected states. If the split of the whole system into a system of interest and its environment is unnecessary, then the evolution of the whole system with Hamiltonian $|H)$ is the process which leads to decoherence, and the preferred basis must depend on such a process. In fact, as the example of Section 3 shows, when the whole system has a Hamiltonian with continuous spectrum:

$$|H)|\omega\rangle = \omega|\omega\rangle, \quad \omega \in [0, \infty), \tag{7.1}$$

the density operator $(\rho_*|$ will be diagonal in the eigenbasis $\{(\omega|, (\omega; \omega'|\}$. This means that $\{(\omega|, (\omega; \omega'|\}$ is the preferred basis: the preferred states are completely stable to the extent that they are time invariant. In other words, the preferred basis is defined by *the eigenstates of the Hamiltonian of the whole system.* From a theoretical point of view, this definition of the preferred basis is more precise than the characterization based on the predictability sieve criterion, which relies on the observer's ability to predict the future evolution of the system of interest: the eigenstates of the total Hamiltonian $|H)$ are necessarily at the top of the predictability sieve as a consequence of their time invariance.

As we have seen, in a general case a CSCO $\{|H), |O_1), \ldots, |O_n)\}$ must be considered. As the result of the further diagonalization of $(\rho_*|$, a new CSCO $\{|H), |P_1), \ldots, |P_n)\}$ emerges: $(\rho_*|$ will be diagonal in the cobasis of states $\{(\omega, p_1, \ldots, p_n|, (\omega, p_1, \ldots, p_n; \omega', p'_1, \ldots, p'_n|\}$. This new CSCO can be called "*preferred CSCO*" to the extent that its eigenvectors define the basis in which $(\rho_*|$ is diagonal. Therefore, in a general case, the preferred basis is the cobasis of states defined by *the eigenstates of the preferred CSCO of the whole system*, to which the Hamiltonian belongs (for details, see Castagnino & Laura, 2000b, Section II-B).[14]

[14] The dependence of the preferred basis on the Hamiltonian's eigenstates does not prevent from localization in different observables when the necessary conditions hold. In particular, localization in position obtains when the interaction Hamiltonian is dominant and it is function of the position operator. In the literature, decoherence is usually studied in measurement situations, where we seek precisely these conditions because we have to read the position of the measurement device's pointer. But this is a

Even though the self-induced approach shows that decoherence depends on the dynamics of a closed system, in order to interpret the results obtained by the einselection approach from the new viewpoint it is necessary to split the whole system into a system of interest and an environment. Under the assumption that the environment is in thermal equilibrium (it does not contribute to the evolution), Paz and Zurek (2000) distinguish three basically different regimes in which the predictability sieve criterion can be successfully applied to select the preferred basis: they differ in the relative strength of the system's self-Hamiltonian and of the interaction Hamiltonian. The first regime is the quantum measurement situation, where the self-Hamiltonian of the system can be neglected and the evolution is completely dominated by the interaction Hamiltonian: in such a case, the preferred states are directly the eigenstates of the interaction Hamiltonian (Zurek, 1981). The second regime is the more realistic and complex situation where neither the self-Hamiltonian of the system nor the interaction with the environment are clearly dominant and both induce non-trivial evolution: in this case, the preferred basis arises from an interplay between self-evolution and interaction; quantum Brownian motion belongs to this case (Paz, 1994). The third regime corresponds to the situation where the dynamics is dominated by the system's self-Hamiltonian: in this case, the preferred states are simply the eigenstates of this self-Hamiltonian (Paz & Zurek, 1998). When these results, obtained case by case by means of the predictability sieve criterion, are considered from the self-induced viewpoint, they turn to be particular cases of a general characterization of the preferred basis: if the preferred states are defined by the eigenstates of the Hamiltonian of the whole system, it is not hard to realize that they will depend on the Hamiltonian's component which dominates the whole evolution.

Perhaps this discussion may seem too technical and lacking conceptual consequences. However, the emergence of the preferred basis is a relevant question when foundational problems are considered. For example, Elby (1994) points out the problems that any basis-selection rule must face in order to determine which observables receive definite values, in particular, the basis degeneracy problem and the imperfect measurement problem. Elby suggests that, apparently, the einselection approach may overcome these problems; he even considers that the main contribution of Zurek's proposal consists in its ability to pick up the preferred basis. If such an opinion is accepted, the seemingly technical discussion acquires a conceptual relevance: the self-induced approach to decoherence supplies a general definition of the preferred basis, not based on an heuristic criterion only applicable case by case.

Finally, it is interesting to dwell on a conceptual point relating with the structure and foundations of quantum mechanics.[15] The fact that the einselection approach works with open systems might be considered as an eloquent manifestation of the contextuality of quantum mechanics, a feature already hinted at by Bohr and later

(*footnote continued*)
particular case of the general phenomenon of decoherence (this is the reason why we prefer to call the preferred basis "preferred" and not "pointer basis").

[15] We thank Dennis Dieks for drawing our attention to this point.

confirmed by many theoretical results. But the close link between a system and its environment is not the only way in which the contextual nature of quantum mechanics becomes manifest. Contextuality is clearly present in the self-induced approach since the system decoheres in the context defined by the preferred CSCO, that is, by the CSCO to which the system's Hamiltonian belongs. In other words, it does not make sense to speak of decoherence independently of a context. This means that the self-induced approach offers a precise definition of what counts as the context in which the system decoheres.

8. Improper mixtures and coarse-graining

Zurek usually describes decoherence as a process which effectively converts quantum entanglement into classical correlations. For him, the incessant "monitoring" of some observables by the environment leads to the "degradation" of pure states into mixtures: "as a result, pure states turn into mixtures and rapidly diagonalize in the einselected states" (Zurek, 2001, p. 13). Such a degradation is manifested by the fact that the purity $\zeta(t)$ of the reduced density operator $\rho_r(t)$ (Eq. (2.14)) decreases during the evolution governed by the generalized master equation. However, as it is well known, according to the unitary Schrödinger (or von Neumann) equation, pure states evolve into pure states: superpositions can never become mixtures. In other words, a unitary evolution cannot make the off-diagonal terms of the density operator vanish. Therefore, how to interpret the non-unitary process that converts a pure state into a mixture? In other words, what is the physical meaning of ρ_r?

Even though Zurek insistently claims that decoherence turns pure states into mixtures, such a claim must be carefully evaluated. On the one hand, ρ_r does not denote a pure state. The fact that the value of its purity $\zeta(t = 0)$ is 1 at the start of the process does not mean that $\rho_r(t = 0)$ can be conceived as a pure state: according to the dynamical equation of quantum mechanics, pure states evolve unitarily preserving its original purity. On the other hand, ρ_r was obtained as a partial trace of the density operator ρ_{SE} corresponding to the composite system; therefore, it does not denote a proper mixture but an *improper mixture*, that is, a mixture obtained by tracing over the degrees of freedom of the system with which the subsystem of interest is entangled (d'Espagnat, 1976). Proper and improper mixtures are represented by the same kind of mathematical object—a density operator; however, they refer to physically different concepts. Let us consider a composite system S, whose components are S_1 and S_2. The state vector space is spanned by the set of product vectors of the form $|a_m\rangle|b_n\rangle$, where $\{|a_m\rangle\}$ is the basis corresponding to S_1 and $\{|b_n\rangle\}$ is the basis corresponding to S_2. If ρ denotes the state of the composite system S, the reduced density operators for S_1 and S_2 are computed as

$$\rho_{r1} = \mathrm{Tr}_2\rho = \Sigma_n \langle b_n|\rho|b_n\rangle, \tag{8.1}$$

$$\rho_{r2} = \mathrm{Tr}_1\rho = \Sigma_m \langle a_m|\rho|a_m\rangle. \tag{8.2}$$

ρ_{r1} and ρ_{r2} are density operators to the extent that they satisfy the normalization condition ($\mathrm{Tr}_1\rho_{r1} = \mathrm{Tr}_2\rho_{r2} = 1$), the Hermitian condition ($\rho_{r1}^{\dagger} = \rho_{r1}$; $\rho_{r2}^{\dagger} = \rho_{r2}$) and the non-negativeness condition ($\langle u|\rho_1|u\rangle \geqslant 0$ and $\langle u|\rho_2|u\rangle \geqslant 0$ for all $|u\rangle$). Moreover, ρ_{r1} and ρ_{r2} are sufficient to calculate the mean value of any observable that belongs exclusively to S_1 or S_2. However, these two reduced density operators are not sufficient, in general, to determine the state of the composite system S, since they provide *no information about the correlations* between the component systems S_1 and S_2. In fact, the state of the composite system S is univocally determined by the reduced density operators of its components only when there are no correlations; in this case, the density operator of S can be expressed as $\rho = \rho_{r1} \otimes \rho_{r2}$. Thus, if we could only make measurements on S_1 and could not make any on S_2, then we should not be able to differentiate the improper mixture denoted by the reduced density operator ρ_{r1} from the analog proper mixture denoted by the density operator ρ_1. But there is no theoretical reason that prevents us from having access to at least some of the traced over degrees of freedom, and this access would permit us to show that the proper mixture and the improper mixture are, in principle, testably different (d'Espagnat, 1995). Nevertheless, this difference seems not to be yet a complete obstacle to consider that the reduced density operator ρ_{r1} denotes the quantum state of S_1: improper mixtures would be the quantum states corresponding to the components of a composite system, even though the quantum state of the composite system would not be uniquely defined by the quantum states of its components (see Hughes, 1989). But this usual opinion forgets the fact that quantum states evolve according the von Neumann equation and, therefore, they always follow unitary evolutions. Improper mixtures, on the contrary, can evolve non-unitarily, that is, they are not subject to the dynamical postulate of quantum mechanics. This means that we cannot conceive reduced density operators as denoting quantum states without rejecting one of the principles of the theory.[16] In other words, in the context of standard quantum mechanics, strictly speaking *improper mixtures are not quantum states*. Therefore, in the case of the einselection approach, ρ_r should not be considered as representing the *quantum state* of the system of interest: the only legitimate quantum state is the state of the whole system environment + system of interest, denoted by $\rho_{SE}(t)$, which evolves unitarily preserving its original purity.

Nevertheless, it is difficult to deny that the reduced density operator ρ_r represents some kind of state to the extent that it supplies, in some sense, a description of the system of interest. The precise sense in which ρ_r provides this description can be explained by means of a generalized conception of coarse-graining. In its traditional form, a coarse-grained description arises from a partition of a phase space into discrete and disjoint cells: this mathematical procedure defines a projector (see Mackey, 1989). In other words, a traditional coarse-graining amounts to a projection

[16] Cartwright (1983) proposed to consider the generalized master equation (with the dissipation term but not yet with the third term associated with decoherence) as the dynamical postulate of quantum mechanics instead of the Schrödinger (or the von Neumann) equation. But who adopts this view is talking about a different theory. Moreover, as we will see, we consider that this move amounts to ignore that there are two different levels of description involved in this discussion.

whose action is to eliminate some components of the state vector corresponding to the fine-grained description: only certain components are retained as "relevant". If this idea is generalized, coarse-graining can be conceived as an operation that reduces the number of components of a generalized vector representing a state. From this viewpoint, taking a partial trace is a particular case of coarse-graining to the extent that a partial trace reduces the number of components of the density operator on which it is applied. In fact, if the density operator ρ of the composite system S mentioned above is expressed as:

$$\rho = \Sigma_{mm'nn'} \rho_{mm'nn'} |a_m\rangle\langle a_{m'}||b_n\rangle\langle b_{n'}|, \tag{8.3}$$

the reduced density operator of S_1 will be an operator that only has components in m, m':[17]

$$\rho_{r1} = \mathrm{Tr}_2\rho = \Sigma_i \langle b_i|\rho|b_i\rangle = \Sigma_{mm'} \rho_{r1mm'} |a_m\rangle\langle a_{m'}|. \tag{8.4}$$

In other words, if the component systems S_1 and S_2 are described by the Hilbert spaces H_1 and H_2, the composite system S corresponds to the Hilbert space $H_1 \otimes H_2$: the coarse-graining resulting from the partial trace $\mathrm{Tr}_2\rho$ consists in considering only the operators of the form $O \otimes I$.

When the notion of coarse-graining is generalized in this sense, it is not difficult to realize that the einselection approach involves a coarse-graining, since the reduced density operator ρ_r is obtained by tracing over the environmental degrees of freedom:[18] ρ_r takes into account only the relevant part of ρ_{SE}, that is, the part considered as "the system", and neglects the irrelevant part associated with "the environment". The example of Section 2 is particularly explicit in this sense: whereas ρ_{SE} has all the components corresponding to the states of the N particles of the environment (see Eq. (2.7), considering (2.5) and (2.6)), ρ_r only has the four components $|s_i\rangle\langle s_j|$, $i, j = 1, 2$, corresponding to the system (see Eq. (2.8), considering (2.9)). This means that the reduced density operator ρ_r supplies a coarse-grained description of the system of interest. In other words, ρ_r does not denote a quantum state but a *coarse-grained state*. Therefore, there is no theoretical reason that prevents it from evolving non-unitarily. On this basis, the evolution of $\rho_r(t)$ turns out to be a situation analogous to the familiar case of unstable dynamical systems, where it is completely natural to obtain a non-unitary coarse-grained evolution from an underlying unitary dynamics. When the coarse-grained nature of ρ_r is accepted, it becomes clear that the claim about pure states turning into mixtures is completely misleading: decoherence is a *coarse-grained process* that results from taking into account only the evolution of some degrees of freedom and disregarding the remaining degrees of freedom conceived as the environment.

[17] $\rho_{r1} = \mathrm{Tr}_2\rho = \Sigma_i\langle b_i|\rho|b_i\rangle = \Sigma_i\langle b_i|(\Sigma_{mm'nn'}\rho_{mm'nn'}|a_m\rangle\langle a_{m'}||b_n\rangle\langle b_{n'}|)|b_i\rangle = \Sigma_{imm'nn'}\rho_{mm'nn'}\delta_{in}|a_m\rangle\langle a_{m'}|\delta_{n'i} = \Sigma_{imm'}\rho_{mm'ii}|a_m\rangle\langle a_{m'}| = \Sigma_{mm'}(\Sigma_i\rho_{mm'ii})|a_m\rangle\langle a_{m'}| = \Sigma_{mm'}\rho_{r1mm'}|a_m\rangle\langle a_{m'}|$.

[18] Anastopoulos (2001, p. 4) notes that the einselection approach involves a coarse-graining because exclusively the operators "that project only on the system's Hilbert space" are considered. However, Anastopoulos does not emphasize that, since taking a partial trace does not amount to a projection, the concept of coarse-graining must be generalized. In fact, we only can assure that $\mathrm{Tr}_\varepsilon\rho_{SE} = \mathrm{Tr}_\varepsilon(\Pi\rho_{SE}\Pi)$, where Π is the projector defined as: $\Pi = \Sigma_i|\varepsilon_i\rangle\langle\varepsilon_i|$.

The self-induced approach to decoherence avoids the possible misunderstandings derived from an inadequate interpretation of the reduced density operator ρ_r, to the extent that such an operator does not take part in the description of the process. In the new context, decoherence is not produced by the interaction between the system of interest and its environment, but results from the own dynamics of the whole quantum system governed by a Hamiltonian with continuous spectrum. Of course, this characterization does not contradict the fact that the off-diagonal terms of a density operator representing a quantum state will never vanish through the unitary evolution described by the von Neumann equation: what decoherence shows is that the mean value $\langle O \rangle_{\rho(t)}$ of any observable $|O) \in V_O^{VH}$ will evolve in such a way that, for $t \to \infty$, it can be computed as the mean value of $|O)$ in a diagonal state $(\rho_*|$. Formally this is expressed by the fact that, even though we can strictly obtain the limit:

$$\lim_{t \to \infty} \langle O \rangle_{\rho(t)} = \langle O \rangle_{\rho_*}, \tag{8.5}$$

the state $(\rho(t)|$ has only a *weak limit* (see Castagnino & Laura, 2000b):

$$W \lim_{t \to \infty} (\rho(t)| = (\rho_*|. \tag{8.6}$$

This weak limit means that, although the off-diagonal terms of the density operator $(\rho(t)|$ never vanish through the unitary evolution, the system decoheres *from an observational point of view*. This means that, even if the system does not decohere for all its possible observables—belonging to the space V_O—, it decoheres from the point of view given by any observable $|O)$ belonging to the van Hove space V_O^{VH}. It is important to stress the role played by $(\rho_*|$ in this explanation of decoherence. The diagonal operator $(\rho_*|$ was not obtained as a partial trace; thus, it can be conceived as representing a quantum state. But $(\rho_*|$ does not denote the real state of the system in the infinite-time limit. The state of the system is represented by $(\rho(t)|$, which does not approach $(\rho_*|$: it always describes an unitary evolution, that is, it does not tend to a definite limit for $t \to \infty$. The only fact that we can strictly assert is that, in the infinite-time limit, the mean value of any observable $|O) \in V_O^{VH}$ can be computed *as if* the whole system were in the state $(\rho_*|$. This means that, in the self-induced approach, the account of decoherence does not involve the evolution of a new kind of state whose interpretation must be supplied. The relevant magnitude is the well known $\langle O \rangle_{\rho(t)}$, which refers to the mean value of $|O)$ in $(\rho(t)|$:[19] this is the magnitude that approaches a definite limit for $t \to \infty$, and no quantum law prevents it from having this kind of behavior.

When decoherence is understood from the new viewpoint, the coarse-grained nature of the process appears in a new light. In fact, $(\rho|O)$ can be thought as representing the state $(\rho|$ of the system "viewed" from the perspective given by the observable $|O)$, for all $|O) \in V_O^{VH}$; in this sense, $\langle O \rangle_\rho$ also involves a sort of coarse-graining. Of course, since in this case we are dealing with continuous variables, we

[19] Let us remember that $\langle O \rangle_{\rho(t)}$ could also be obtained in the Heisenberg picture, where the observable $|O)$ evolves in time whereas the state $(\rho|$ is constant; in this case, we would obtain a diagonal operator $|O_*)$. This fact clearly shows that the fundamental magnitude in the explanation of decoherence is $\langle O \rangle_{\rho(t)}$ and not $(\rho_*|$.

cannot strictly speak of reducing the number of components of a vector state. However, the action of the functional $(\rho|$ onto the observable $|O)$ can be characterized in terms of a generalized notion of projection, which permits $\langle O \rangle_\rho$ to be conceived as the result of a projection of the state $(\rho|$. In fact, we can define a projector belonging to the space $V_O^{VH} \otimes V_S^{VH}$:

$$\Pi = |O)(\rho_O|, \tag{8.7}$$

where $(\rho_O| \in V_S^{VH}$ satisfies[20]

$$(\rho_O|O) = 1. \tag{8.8}$$

In this case

$$(\rho_{rel}| = (\rho|\Pi = (\rho|O)(\rho_O|, \tag{8.9}$$

where $(\rho_{rel}|$ is the projected part of $(\rho|$, relevant for decoherence. This means that $\langle O \rangle_\rho = (\rho|O)$ is the projection of $(\rho|$ onto a subspace of V_S^{VH}; such a subspace is defined by a state $(\rho_O|$, corresponding to the observable $|O)$. On this basis we can understand why $\langle O \rangle_\rho$ can be conceived as a coarse-grained magnitude, that gives us the partial description of $(\rho|$ from the perspective given by $|O)$. This shows, from a different viewpoint, that decoherence is a coarse-grained process, resulting in this case from the coarse-graining introduced by the observable of interest on the underlying unitary dynamics.

The arguments presented in the previous paragraphs lead us to a further conclusion: if we accept that decoherence is a relevant element for explaining how the classical world arises from the quantum realm, we must recognize that classicality is an emergent property belonging to a coarse-grained level of description. But, at this point, a new objection to the decoherence program may be posed: if decoherence leads to classicality only in a coarse-grained sense, then our classical world is a subjective appearance only due to our limited access to reality. The opinion that the einselection approach is subjective or even anthropocentric has been sustained by many authors. For example, Zeh (1971) acknowledges an element of subjectivity in the arbitrariness of the separation of the Universe into subsystems. For Stamp (1995), decoherence involves the same kind of subjectivity as irreversibility: it only supplies a FAPP ("for all practical purposes") solution of the problem of the transition from quantum to classical. A similar position is adopted by d'Espagnat (1995) when he asserts that the einselection approach only supplies a consistent description of the "empirical" reality, but it cannot be considered as providing an objective description of reality in a strong ontological sense. Of course, a full treatment of this matter would require a careful philosophical discussion which largely exceeds the purposes of the present paper. Nevertheless, some conceptual points may be emphasized. In the first place, since the self-induced approach does not introduce assumptions regarding physical interactions with the observer, it is more immune than the einselection approach to the charge of subjectivity. The perception of classicality, explained by Zurek on the basis of the stable correlations

[20] This condition guarantees that: $\Pi^2 = |O)(\rho_O|O)(\rho_O| = |O)(\rho_O| = \Pi$.

among the states of the observer's brain, the observed system and the environment, requires a strong interaction completely absent in the new approach which only relies on a coarse-grained description: coarse-graining is a theoretical descriptive operation, not the result of a physical interaction. It is in this sense that we claimed that the self-induced approach is a relevant element for explaining the emergence of classicality and not our perception of classicality. However, the charge of subjectivity may be carried one step further by arguing that coarse-graining, by itself, also introduces an element of subjectivity into the theory. This leads us to a second conceptual point, which reappears in many fields of physics, such as statistical mechanics with regard to irreversibility or unstable dynamics with regard to indeterminism: all these cases involve a coarse-grained description whose objectivity is traditionally questioned. Nevertheless, this traditional position may be challenged by adopting Putnam's ontological pluralism (Putnam, 1981), according to which ontology arises from the synthesis between our conceptual scheme and the noumenal realm; therefore, different objective descriptions of reality may coexist to the extent that each one of them "cuts out" its own ontology from the same noumenal substratum. Certainly, these brief remarks do not amount to a full argument (for a detailed discussion, see Lombardi, 2002), but they suggest that it is possible to argue that coarse-graining does not imply subjectivity but constitutes a new ontology: a coarse-grained description is not a mere subjective perspective but a different way of describing reality, a way no less objective than the descriptions given by the so-called "fundamental" theories. Nevertheless, it is worth while to remember that this is not a problem derived from the peculiar features of the self-induced approach to decoherence: as we have seen, the einselection approach also introduces a coarse-graining when the partial trace is applied. This means that, to the extent that decoherence involves coarse-graining in one or another way, the decoherence theorist must face the challenge of explaining the objective character of the classical world.

9. The emergence of classicality

At this point it is convenient to stress that here we are not arguing that the self-induced approach provides a full account of the emergence of classicality: our central purpose was to present a new approach to decoherence and to compare it with the mainstream einselection approach. Nevertheless, it is interesting to note that the self-induced perspective also avoids some difficulties derived from certain criticisms that have been directed to the decoherence program in its einselection version.

Zurek's program has been objected by appealing to the argument that decoherence does not explain the occurrence of a particular event corresponding to a definite value of an observable (see e.g. Adler, 2003): the fact that the reduced density operator ρ_r becomes diagonal for $t \to \infty$ does not tell us yet which particular event turns out to be actual. Zurek tries to overcome this difficulty by means of his "existential interpretation" of quantum mechanics (Zurek, 1998), based on Everett's relative state interpretation: the events associated with the diagonal elements of ρ_r occur in a sense relative to the observer's memory. Some authors consider that the appeal to Everett's

interpretation supplies the answer to the problem of interpreting ρ_r (see e.g. Bacciagaluppi & Hemmo, 1994). However, Zurek's interpretative move does not provide an acceptable solution to the problem: as Bub (1997) claims, the reduced density operator ρ_r not only fails to account for the occurrence of *just one* of the events associated with the definite values of the observable of interest, but it is actually *inconsistent* with such an occurrence. This means that ρ_r in the infinite-time limit:

$$\rho_r = |\alpha|^2 |s_1\rangle\langle s_1| + |\beta|^2 |s_2\rangle\langle s_2| \tag{9.1}$$

cannot be interpreted as describing the occurrence of a particular event, either the event associated with $|s_1\rangle$ or the event associated with $|s_2\rangle$, with the coefficients $|\alpha|^2$ and $|\beta|^2$ representing a measure of our ignorance about the actual event, derived from ignoring—tracing over—the environmental degrees of freedom: taking full account of the environment gives us back the pure state ρ_{SE} from which ρ_r was derived. This fact becomes clearer when the coarse-grained origin of the reduced density operator ρ_r is recognized: when we "remove" the coarse-graining, we recover the underlying state with its unitary evolution; but, in the quantum case, this removal manoeuvre does not return us the precise value of the observable of interest as in the classical case. It is not difficult to conjecture that Zurek's misguided insistence on considering ρ_r as a quantum mixture—which, thereby, can be interpreted in terms of ignorance—is the consequence of disregarding the coarse-grained origin of ρ_r.

The self-induced approach sidesteps the discussions derived from seeking an adequate interpretation of a new kind of state denoted by the reduced density operator ρ_r, which cannot be given an ignorance interpretation: such an operator is not involved in the explanation of the phenomenon of decoherence provided by the new perspective. The relevant magnitude in the self-induced approach is the well known mean value $\langle O\rangle_{\rho(t)}$: the point is that, in the infinite-time limit, such a mean value settles down in a value that can be computed as if the system were in the diagonal quantum state $(\rho_*|$. The acknowledgement of the coarse-grained nature of the process of decoherence avoids, from the very beginning, the temptation of interpreting an operator resulting from a partial trace in terms of ignorance as if it were a quantum mixture.

What the self-induced approach states is that quantum correlations vanish when the system is described from a coarse-grained viewpoint given by the observable of interest. This means that, in the descriptive level defined by such a viewpoint, the system's description looses its non-Boolean character, which represents the main obstacle for an adequate explanation of the behavior of the classical world. Therefore, the fact of obtaining a descriptive level where quantum correlations have vanished seems to be a relevant step towards the account of the emergence of classicality.[21] In particular, the explanation of the emergence of classicality in quantum measurements involves two profoundly different stages: the first one is to show how and under what conditions quantum correlations tend to vanish; the

[21] Some further steps have been made in this direction. For instance, in Castagnino and Laura (2000b) and Castagnino and Lombardi (2002) it is shown that, when $(\rho_*|$ is Wigner-transformed, for $h \to 0$ it is resolved into a sum of classical trajectories weighted by well defined probabilities.

second one is to explain the occurrence of a particular value of an observable on a given run of the experiment. Of course, the mean value of the observable of interest does not explain yet the occurrence of the event corresponding to a particular value of this observable: such an explanation requires to supply an interpretation of $\langle O \rangle_{\rho(t)}$. For the ensemble interpretation of quantum mechanics, for instance, the mean value of an observable is a basic and meaningful magnitude since the theory refers to ensembles of similarly prepared systems. Who rejects this position is committed to provide an interpretation of the probabilistic notions involved in the definition of the mean value. But these interpretative questions do not arise from the decoherence program in its self-induced version: they are questions that must be faced in any case when the general problem of the interpretation of quantum mechanics is considered. These are matters that exceed the purposes of the present paper since, as we have said from the beginning, problems of interpretation are not our subject here. Our remarks in this section are only directed to stress that the self-induced approach, by itself, provides neither a *full* account of the transition from quantum to classical nor a *complete* solution of the measurement problem, to the extent that both questions require to take a position with regard to the whole interpretation of quantum mechanics.

10. Conclusions

During the last decades, an impressive number of results have been obtained by means of the einselection approach to decoherence; such results are extremely valuable from the viewpoint of physicists' everyday activity. In this sense, the works of Zurek and his collaborators represent an outstanding contribution to physics. However, the traditional approach leads to certain conceptual difficulties when applied to foundational problems. The self-induced approach tries to overcome some of these difficulties by means of a general and mathematically well founded account of decoherence, according to which the phenomenon of decoherence is the result of the dynamics of a closed quantum system governed by a Hamiltonian with continuous spectrum.

In this paper, our aim was to compare this approach with the traditional one, in order to point out some conceptual advantages of the new way of conceiving decoherence. We have stressed that this decoherence program does not provide yet a full explanation of quantum measurement nor of the transition from quantum to classical. Nevertheless, we agree with those who consider that decoherence must be taken into account in the treatment of these problems. If this opinion is accepted, then it seems reasonable to pay attention to a new proposal in the task of seeking an answer of questions that continue to be unsolved up to the present.

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